(1) [15 pts] Determine whether each of the following statements are true or false. Justify your answer using a short proof or counterexample.
   (a) If $\hat{\theta}$ is an unbiased estimator for $\theta$, it is also consistent.
   (b) Let $X_1, \ldots, X_n$ be a random sample where $X_i \sim p(x_i | \theta)$, where $s(x_1, \ldots, x_n)$ is a minimal sufficient statistic of size $k < n$. If the maximum likelihood estimator exists and is unique, the likelihood ratio statistic to contrast $H_0 : \theta = \theta_0$ vs. $H_a : \theta \neq \theta_0$ depends on the data only through the sufficient statistic.
   (c) Let $X_1, \ldots, X_n$ be a random sample where $X_i \sim p(x_i | \theta)$ and $I(\theta) = n/\theta^2$ be the Fisher’s information for $\theta$. If we define $\phi = \theta^2$, then Fisher’s information for $\phi$ is $I(\phi) = n/\phi$.

(2) [30 pts] Consider two random samples $X_1, \ldots, X_{n_1}$ and $Y_1, \ldots, Y_{n_2}$ arising from two Bernoulli distributions with parameters $p_1$ and $p_2$ respectively. Determine the maximum likelihood estimator for $(p_1, p_2)$ if we know that $0 \leq p_1 \leq p_2 \leq 1$. Are the resulting estimators unbiased? Are they consistent?

(3) [35 pts] Consider two random samples $X_1, \ldots, X_{n_1}$ and $Y_1, \ldots, Y_{n_2}$ with $X_i \sim \text{Uni}[-\theta_1, \theta_1]$ and $Y_i \sim \text{Uni}[-\theta_2, \theta_2]$. Define the likelihood ratio test in this problem. Is it true that as $n_1, n_2 \to \infty$ the likelihood ratio $\Lambda$ converges to a $\chi^2_1$ distribution?

(4) [20 pts] Let a random sample of size $n$ be taken from an exponential distribution with expectation equal to $\theta$. Find the MVUE of $P(X \leq 2)$. 