1. Let us assume that the life of a tire in miles, say $X$, is normally distributed with mean $\theta$ and standard deviation 5000. Past experience indicates that $\theta = 30,000$. The manufacturer claims that the tires made by a new process have mean $\theta > 30,000$, and it is very possible that $\theta = 35,000$. Let us check his claim by testing $H_0 : \theta = 30,000$ against $H_1 : \theta > 30,000$. We shall observe $n$ independent values of $X$, say $x_1, \ldots, x_n$, and we will reject $H_0$ (thus accept $H_1$) if and only if $\bar{x} \geq c$. How large a sample $n$ is necessary if we set $c = 33,000$ and want the power of the test to be 0.98 when the true value is $\theta = 35,000$. What is the size of this test?

2. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size $n = 4$ from a distribution with pdf $f(x; \theta) = \frac{1}{\theta} I_{0 < x < \theta}$, where $0 < \theta$. The hypothesis $H_0 : \theta = 1$ is rejected and $H_1 : \theta > 1$ is accepted if the observed $Y_4 > c$.

(a) Find the constant $c$ so that the significance level is $\alpha = 0.05$.
(b) Find the power function of the test.

3. Assume that the weight of cereal in a “10-ounce box” is $N(\mu, \sigma^2)$. To test $H_0 : \mu = 10.1$ against $H_1 : \mu > 10.1$, we take a random sample of size $n = 16$ and observe that $\bar{x} = 10.4$ and $s = 0.4$.

(a) Do we accept or reject $H_0$ at the 5 percent significance level?
(b) What is the approximate $p$-value of this test?

4. Among the data collected for the World Health Organization air quality monitoring project is a measure of suspended particles in $mcg/m^3$. Let $X$ and $Y$ equal the concentration in $mcg/m^3$ of suspended particles in the city center (commercial district) for Melbourne and Houston, respectively. Using $n = 13$ observations of $X$ and $m = 16$ observations of $Y$, we shall test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X < \mu_Y$. Let $\alpha = 0.05$.

(a) Define the test statistic and critical region, assuming the variances are equal.
(b) If $\bar{x} = 72.9$, $s_x = 25.6$, $\bar{y} = 81.7$, and $s_y = 28.3$, calculate the value of the test statistic and state your conclusion.

5. Mendelian theory states that the number of a certain type of pea falling into the classifications “round and yellow”, “wrinkled and yellow”, “round and green”, and “wrinkled and green” should be in the ratio 9:3:3:1 (i.e., 9/16 of the peas should be round and yellow). Suppose that 100 such peas are gathered and it is found that there are 56, 19, 17, and 8 in the respective categories. Are these data consistent with the model? Use $\alpha = 0.05$.

6. Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution with mean zero and variance $\theta$, $0 < \theta < \infty$. Show that $\sum_{i=1}^{n} X_i^2 / n$ is an unbiased estimator of $\theta$ and has variance $\frac{2\theta^2}{n}$.

7. Prove that the sum of the observations of a random sample of size $n$ from a Poisson distribution having parameter $\lambda$, $0 < \lambda < \infty$, is a sufficient statistic for $\lambda$.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n$ from a beta distribution with parameters $\alpha = \theta > 0$ and $\beta = 2$. Show that the product $X_1 X_2 \cdots X_n$ is a sufficient statistic for $\theta$. 
