1. Let $X_1, \ldots, X_n$ be a random sample from each of the distributions having the following probability density functions:

(a) $f(x; \theta) = \theta x^{(\theta-1)}, 0 < x < 1, 0 < \theta < \infty$, zero elsewhere

(b) $f(x; \theta) = \frac{1}{2} \exp(-|x - \theta|)$, where the ranges of $x$ and $\theta$ are the real line

In each case, find an estimator of $\theta$ by the method of moments and show that it is consistent (hint: to show consistency, consider using Slutsky’s theorem and the fact that if a sequence converges in distribution to a constant, then it converges in probability to that constant).

2. Let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of a random sample of size $n$ from the uniform distribution of the continuous type over the closed interval $[\theta - \rho, \theta + \rho]$. Find the maximum likelihood estimators for $\theta$ and $\rho$. Are these two unbiased estimators?

3. Let the observed value of the mean $\bar{X}$ of a random sample of size 18 from a distribution that is $N(\mu, 80)$ be 81.6. Find a 95% confidence interval for $\mu$.

4. Let a random sample of size 17 from the normal distribution $N(\mu, \sigma^2)$ yield $\bar{x} = 4.7$ and $s^2 = 5.76$. Determine a 90 percent confidence interval for $\mu$.

5. Suppose it is known that a random variable $X$ has a Poisson distribution with parameter $\lambda$. A sample of 200 observations from this population has a mean equal to 3.4. Construct an approximate 90% confidence interval for $\lambda$.

6. Let two independent random samples, each of size 10, from two normal distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\bar{x} = 4.8, (s_1)^2 = 8.64$, $\bar{y} = 5.6, (s_2)^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$. 