Quizz 1: 1D continuous systems: bifurcations (part 1)

Choose one of the two following dynamical systems, and study it giving as much information as possible. Both questions are worth the same amount of points. Always justify your answers.

**SYSTEM 1:** \( \dot{x} = r + x - \ln(1 + x) \)

**SYSTEM 2:** \( \dot{x} = x - rx(1 - x) \)

You may consider the following questions:

- if analytical solutions for the fixed point(s) exist, write them explicitly. If they don’t, plot the relevant phase portraits indicating the various possible cases.
- determine the stability of the fixed point(s) (using whichever method you prefer).
- find the critical value(s) of the parameter \( r_* \), and the corresponding fixed point \( x_*(r_*) \) for which a bifurcation occurs.
- determine the type of bifurcation.
- draw a quantitatively accurate bifurcation diagram, making sure to indicate all of the information above.
- reduce the system to its normal form near the bifurcation.