I have not included any theoretical exercise about distributions in this homework. My goal is to help you recognize situations where some of the special distributions can be helpful and practice some of the basic tools you learned in AMS-131, like conditional probabilities, expectations and variances. All problems are taken from, or modified from problems in DGS, third edition.

(1) The probability that each specific child in a given family will inherit a certain disease is \( p \). In a family with \( n \) children,
   (a) What is the probability that \( k \) children inherit the disease? What is the expected number of children with the disease? What is the variance of the number of children with the disease?
   (b) If it is known that the firstborn child in the family has inherited the disease, what is the probability that a total of \( k \) children inherit it?
   (c) If it is known that at least one child in the family has inherited the disease, what is the expected number of children in the family who have inherited it?

(2) Suppose that two players \( A \) and \( B \) are trying to throw a basketball through a hoop. The probability that player \( A \) will succeed on any given throw is \( p \) and he throws until he has succeeded \( r \) times. The probability that player \( B \) will succeed on any given throw is \( mp \), where \( m \) is an integer greater or equal than 2 such that \( mp < 1 \); and she throws until she has succeeded \( mr \) times.
   (a) What is the probability that player \( A \) needs \( s \) throws to get the first ball through the hoop?
   (b) What is the probability that player \( A \) makes a total of \( d \) throws?
   (c) For which player is the expected number of throws smaller?
   (d) For which player is the variance of the number of throws smaller?

(3) Suppose that \( n \) students are selected at random without replacement from a class containing \( T \) students, of whom \( A \) are boys and \( T - A \) are girls. Let \( X \) denote the number of boys that are obtained. For what sample size \( n \) will \( \text{Var}(X) \) be maximum?

(4) Suppose that a book with \( n \) pages contains on the average \( \lambda \) misprints per page.
   (a) What assumptions make it reasonable to assume that the number of mistakes follows a Poisson process?
   (b) What is the probability that a particular page will contain no misprints?
   (c) What is the expected number of misprints and the variance of the number of misprints in \( m \) pages?
   (d) What is the probability that there will be at least \( m \) pages which contain more than \( k \) misprints?

(5) Suppose that 16 percent of the students in a certain high school are freshmen, 14 percent are sophomores, 38 percent are juniors and 32 percent are seniors. If 15 students are selected
at random from the school, what is the probability of observing 4 freshmen, and at least 6 sophomores or juniors?. Justify any assumption you use in solving this problem. You can leave the expression in terms of a sum.

(6) Suppose that on a certain examination in advanced mathematics, students from university $A$ achieve scores that are normally distributed with a mean of 625 and a variance of 100, and students from university $B$ achieve scores which are normally distributed with a mean of 600 and a variance of 150. If two students from university $A$ and three students from university $B$ take this examination, what is the probability that the average of the scores of the two students from university $A$ will be greater than the average of the scores of the three students from university $B$?

(7) Suppose that the measurement $X$ of pressure made by a device in a particular system has a normal distribution with mean $\mu$ and variance 1, where $\mu$ is the true pressure. Suppose that the true pressure is unknown but has a uniform distribution on the interval $[5, 15]$.
(a) What is the probability that a pressure measurement will be greater than 8?
(b) What is the conditional p.d.f. of $\mu$ given one single measurement, $X = 8$.

(8) The lifetime of a light bulb is exponential with mean lifetime of 1000 hours. A second spare bulb (with the same characteristics) is available to be installed if the first one fails.
(a) What is the probability that the first bulb will last at least 1000 hours?
(b) What is the probability that the first bulb will last at least 2000 hours if has already lasted 1000 hours?
(c) What is the probability that both bulbs will burn before 1000 hours?

(9) Suppose that an electronic system contains $n$ similar components that function independently of each other. Suppose also that the length of life of each component, measured in hours, has a Weibull distribution distribution with parameter $a$ and $b$.
(a) If the components are connected in series, what is the probability that the system will work for $s$ hours before failing? What is the expected value of the lifetime of the system?
(b) What if the components are connected in parallel?

(10) Suppose that the proportion $X$ of defective items in a large lot is unknown, and $X$ has a beta distribution with parameters $\alpha$ and $\beta$. If one item is selected at random from the lot, what is the probability that it will be defective?