ENG-27: Homework 2

1. Find, where they exist, the matrices, $A + B$, $A - B$, $AB$, $BA$, $(A + B)(A - B)$ and $A^2 - B^2$, if

\[
(i) \quad A = \begin{pmatrix} 4 & -5 \\ 2 & 0 \\ -6 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 7 \\ 2 & -5 \end{pmatrix},
\]

\[
(ii) \quad A = \begin{pmatrix} 3 & -4 \\ 1 & 5 \\ -2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 0 & 2 \end{pmatrix}
\]

and

\[
(iii) \quad A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 3 & -5 \\ 2 & 0 & 6 \\ 1 & -1 & 4 \end{pmatrix},
\]

2. If

\[
A = \begin{pmatrix} 1 & 7 & 5 \\ -1 & 4 & 0 \\ 6 & 3 & 1 \end{pmatrix},
\]

evaluate $A + A^T$ and $A - A^T$. Show that $A + A^T$ is symmetric and $A - A^T$ is skew-symmetric. Is this a general result for square matrices?

3. If

\[
A = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix},
\]

show that $(A - I)^2 = 0$ (with $I$ and 0 the identity and zero matrices of order $2 \times 2$).

4. Show that

\[
\begin{vmatrix} 4 & -3 & 1 \\ 7 & 2 & -2 \\ 1 & -5 & 3 \end{vmatrix} = 16,
\]

by (a) directly expanding along the first row, and (b) using as many column and row operations as you can.

5. Verify that the matrix,

\[
M = \frac{1}{3} \begin{pmatrix} 1 & 2\sqrt{2} \\ -1 & 2 \end{pmatrix}
\]

is orthogonal (meaning that $M^{-1} = M^T$), and show that $\det(M) = 1$.

6. Show that

\[
\begin{vmatrix} t + 3 & -1 & 1 \\ 5 & t - 3 & 1 \\ 6 & -6 & t + 4 \end{vmatrix} = (t + 2)(t - 2)(t + 4),
\]

where $t$ is a parameter. Hence give the values of $t$ for which the corresponding matrix is singular.