PRACTICE EXAM 2
11-16 February 2011

Instructions

• Please turn your OFF your cell phone, iPod, etc..

• There are 4 questions worth a total of 40 points. 100%=40 points.

• Please write neatly and show all your work. You do not need to simplify your answers, unless a numerical value is called for.

• No notes or books.

• NO calculators, NO PDAs, etc.

• Take your time, and check your answers.

Use the Method of Thinking!! And joy of the day to you

NAME: ________________________________

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Problem 1. Chops: Each question is worth 1 point. **No partial credit given.**

A) If \( f(x, y) = \ln(4x^9 + y^5) \) what is \( f_y \)?

B) If \( f(x, y) = \ln(4x^9y^5) \) what is \( f_x \)?

C) If \( f(x, y) = \exp(4x^9y^5) \) what is \( f_x \)?

A bird foraging for \( t \) minutes at Younger Lagoon gets energy gain, measured in kcal, \( G(t) = 60\frac{t}{t+12} \).

D) What are the units of 60 on the right hand side?

F) How long must the bird forage to get 40 kcal?

G) Find the first three derivatives of \( f(z) = \frac{1}{1+2z} \) without using the quotient rule.

J) If \( f(x, y) = 4\ln[(4x + y)^{-2}] \) what is \( f_{yyx} \)?

K) If \( f(x, y) = 3^{xy} \) what is \( f_y \). It may be helpful to recall that: \( 3^z = e^{\ln(3^z)} \)
L) If \( f(x, y) = 6xe^{2xy} \) what is \( f_x \)?

M) If \( g(w, z) = 28z\ln(4z + w) \) what is \( g_z \)?
Problem 2 A) (5 pts) Find the critical point(s) of $f(x, y) = x^2 + 3xy + y^2 - 9x - 11y + 30$

B) (5 pts) Find and classify the critical points of $f(x, y) = \ln(xy) + 2x^2 - xy - 6x$
Problem 3 A manufacturer produces two goods, A and B, the joint cost function.

\[
c(q_A, q_B) = 1.5q_A^2 + 3q_B^2
\]

The demand functions for the products are \( p_A = 60 - q_A^2 \) and \( p_B = 72 - 2q_B^2 \).

A) (2 pts) What is the profit function?

B) (8 pts) What level of production maximizes profit?
Problem 4 A consumer has allotted $I$ dollars a day to pay for coffee (mainly lattes), $c$, and power bars $p$. Lattes cost $3.00 each and power bars $2.00 each. His daily utility function for these is

$$U(c, p) = 2\ln(c + 1) + 4\ln(p + 1)$$

A) (2 pts) What is the relationship between his allocation and his consumption of power bars and coffee??

B) (2 pts) Suppose that he decides to limit himself to one power bar a day. Sketch $U(c, 1)$.

C) (2 pts) Interpret your sketch from part B.

D) (2 pts) What are the rates of consumption of coffee and power bars that maximize his utility? (You may assume that the critical point is a maximum.)
E) (2 pts) Suppose that he decides to increase his allocation for these items by $2 a day. Approximately how much will his optimal utility increase?
Problem 1. Chops: Each question is worth 1 point. No partial credit given.

A) If \( f(x, y) = \ln(4x^9 + y^5) \) what is \( f_y \)?

\[
 f_y = \frac{1}{4x^9 + y^5} \cdot 5y
\]

B) If \( f(x, y) = \ln(4x^9y^5) \) what is \( f_x \)?

\[
 f(x, y) = \ln(4x^9y^5) = \ln(4x^9) + \ln(y^5)
\]
so \( f_x = 9 \frac{x}{x} \)

C) If \( f(x, y) = \exp(4x^9y^5) \) what is \( f_x \)?

\[
 f_x = \exp[4x^9y^5] \cdot 36x^8
\]

A bird foraging for \( t \) minutes at Younger Lagoon gets energy gain, measured in kcal, \( G(t) = 60 \frac{t}{t+12} \).

D) What are the units of 60 on the right hand side?

\( \text{kal} \)

E) How long must the bird forage to get 40 kcal?

\[
40 = 60 \cdot \frac{t}{t+12} \Rightarrow \frac{2}{3} = \frac{t}{t+12} \Rightarrow 0 = 4 \Rightarrow \text{no credit, no units}
\]

F) Find the first three derivatives of \( f(z) = \frac{1}{1+2z} \) without using the quotient rule.

\[
 f(z) = (1+2z)^{-1} \quad f'(z) = -2(1+2z)^{-2} \quad f''(z) = 8(1+2z)^{-3}
\]

G) Find the first three derivatives of \( f(z) = 4\ln[(4z + y)^{-2}] \) what is \( f_{yy}z \)?

\[
 f(z) = -8\frac{\ln(4x+y)}{(4x+y)^2} \quad f_{yy} = -\frac{16}{(4x+y)^3}
\]

H) If \( f(x, y) = 3^{xy} \) what is \( f_y \). It may be helpful to recall that: \( 3^z = e^{\ln(3^z)} \)

\[
 f(x, y) = 3^{xy} = e^{xy \ln(3)} \quad f_y = e^{xy \ln(3)} \times \ln(3) = \frac{1}{3} \]

\[
 f(x, y) = e^{xy \ln(3)} \times \ln(3) = \frac{1}{3} \]

\[
 f(x, y) = e^{xy \ln(3)} \times \ln(3) = \frac{1}{3} \]
L) If $f(x, y) = 6xe^{2xy}$ what is $f_x$?

$$f_x = 6e^{2xy} + 6xe^{2xy} \cdot 2y$$

M) If $g(w, z) = 28z\ln(4z + w)$ what is $g_z$?

$$g_z = 28\frac{\ln(4z + w)}{4z + w} + \frac{28z}{4z + w} \cdot 4$$
Problem 2 A) (5 pts) Find the critical point(s) of \( f(x, y) = x^2 + 3xy + y^2 - 9x - 11y + 30 \)

\[
\begin{align*}
\frac{f_x}{f_y} &= \begin{cases} 2x + 3y - 9, & \text{if } \frac{f_x}{f_y} = 0 \Rightarrow (2x + 3y - 9) \times 2 \\
3x + 2y - 11, & \text{if } (3x + 2y - 11) \times 3 \\
\Rightarrow 4x + 6y = 18 \\
9x + 6y = 33 \\
\text{Subtract} \Rightarrow 5x = 15 \\
\Rightarrow x = 3 \\
(3, 1) \text{ is the only critical point}
\end{cases}
\end{align*}
\]

B) (5 pts) Find and classify the critical points of \( f(x, y) = \ln(xy) + 2x^2 - xy - 6x \)

\[
\begin{align*}
\frac{f_x}{f_y} &= \begin{cases} \frac{1}{x} + 4x - y - 6, & \text{if } f_x = 0 \Rightarrow \frac{1}{x} + 4x - 1 \times 6 = 0 \\
\frac{1}{y} - x, & \text{if } \frac{1}{y} - x = 1 \\
\frac{f_{xx}}{f_{yy}} &= \begin{cases} -\frac{1}{x^2} + 4, & \text{if } x = 3, 2 \frac{1}{3} \Rightarrow (3, 2 \frac{1}{3}) \text{ is the only critical point} \\
-\frac{1}{y^2} & \text{if } y = 2 \frac{1}{3} \\
f_{xy} &= -1 \\
D &= \left[ 4 - \frac{1}{(3 \frac{1}{3})^2} \right] \left[ -\frac{1}{(2 \frac{1}{3})^2} \right] - (-1)^2 \\
&= \left[ 4 - \frac{4}{9} \right] \left[ -\frac{9}{4} \right] - 1 \\
&= -1 \\
&\Rightarrow D < 0 \text{ saddle point, neither max nor min}
\end{cases}
\end{align*}
\]
Problem 3 A manufacturer produces two goods, A and B, the joint cost function.

\[ c(q_A, q_B) = 1.5q_A^2 + 3q_B^2 \]

The demand functions for the products are \( p_A = 60 - q_A^2 \) and \( p_B = 72 - 2q_B^2 \).

A) (2 pts) What is the profit function?

\[
\pi(q_A, q_B) = q_A (60 - q_A^2) + q_B (72 - 2q_B^2) - (1.5q_A^2 + 3q_B^2)
\]

\[ = 60q_A - q_A^3 + 72q_B - 2q_B^3 - 1.5q_A^2 - 3q_B^2 \]

B) (8 pts) What level of production maximizes profit?

\[
\frac{\partial \pi}{\partial q_A} = 60 - 3q_A^2 - q_A^2 = 0
\]

\[ = 3 [20 - q_A^2 - q_A^2] = 6 [12 - 6q_A^2 - q_B^2] \]

\[ = 36 [10 + 6q_A][1 - q_A] = 6 [4 + 6q_B][3 - q_B] \]

For \( \frac{\partial \pi}{\partial q_A} = 0 \) \( \Rightarrow q_A^* = 4 \) \( q_B^* = 3 \) \( \text{[demand > 0]} \)

\[
\frac{\partial^2 \pi}{\partial q_A^2} = -6q_A - 2 \quad \frac{\partial^2 \pi}{\partial q_B^2} = -12q_B - 6 \quad \frac{\partial^2 \pi}{\partial q_A \partial q_B} = 0
\]

\[ \det D = [-27][-42] - [0]^2 > 0 \]

\[ \text{and } \frac{\partial^2 \pi}{\partial q_A^2} < 0 \Rightarrow (4, 3) \text{ maximizes profit} \]
Problem 4 A consumer has allotted $I$ dollars a day to pay for coffee (mainly lattes), $c$, and power bars $p$. Lattes cost $3.00 each and power bars $2.00 each. His daily utility function for these is

$$U(c, p) = 2\ln(c + 1) + 4\ln(p + 1)$$

A) (2 pts) What is the relationship between his allocation and his consumption of power bars and coffee?

$$3c + 2p = I$$

B) (2 pts) Suppose that he decides to limit himself to one power bar a day. Sketch $U(c, 1)$.

C) (2 pts) Interpret your sketch from part B.

Coffee has increasing utility at a decreasing rate.

D) (2 pts) What are the rates of consumption of coffee and power bars that maximize his utility? (You may assume that the critical point is a maximum.)

$$\hat{U}(c, p, \lambda) = 2\ln(c+1) + 4\ln(p+1) - \lambda \left[ 3c + 2p - I \right]$$

$$\hat{U}_c = \frac{2}{c+1} - 3\lambda = 0 \Rightarrow \lambda = \frac{2}{3(c+1)}$$

$$\hat{U}_p = \frac{4}{p+1} - 2\lambda = 0 \Rightarrow \lambda = \frac{2}{p+1}$$

Apply constraint

$$3c + 2(3c+2) = I$$

$$9c = I - 4$$

$$c^* = \frac{I - 4}{9}$$

$$p^* = 3c^* + 2 = \frac{I - 4}{3} + 2$$
E) (2 pts) Suppose that he decides to increase his allocation for these items by $2 a day. Approximately how much will his optimal utility increase?

\[ \frac{\partial U^*}{\partial I} = \lambda^* \]

Now \( \lambda^* = \frac{2}{\frac{I}{3} + (I-4 + \phi) + 2 + 1} = \frac{6}{I-4+\phi} = \frac{6}{I+5} \)

\[ I \rightarrow I + 2 \leq (\Delta I = 2) \]

\[ \Delta U^* = \lambda^* \Delta I = \left( \frac{6}{I+5} \right) 2 = \frac{12}{I+5} \]