7 JAN 2018

HW is due on Webs before lecture starts

\( z = f(x, y) \)

\[
\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}
\]

\[
\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}
\]

\( f(x, y) = e^{6x+3y} \)

Recall: \( \frac{d}{dx} e^x = e^x \)

\[
\frac{d}{dx} e^{g(x)} = \frac{d}{dx} h(g) = h(g) \cdot e^{g(x)}
\]
\[ h(y) = e^y \]

\[ \frac{d}{dx} e^{g(x)} = \frac{d}{dy} \frac{dg}{dx} = e^y \frac{dg}{dx} \]

\[ \frac{d}{dx} e^{g(x)} = e^{g(x)} \frac{dg}{dx} \]

\[ f(x, y) = e^{6x+3y} \]

Find \( f_x \), note \( e^{6x+3y} = e^6 \cdot e^y \).

So \( f_x = 6e^6 \cdot e^y = 6e^y \)

Or just do \( f_x = e^{6x+3y} \frac{d}{dx} [6x+3y] \):

\[ = e^{6x+3y} \cdot 6 \]

\[ = 6e^{6x+3y} \]

\[ = 6e \]
\[ f(x, y) = e^{x(1+y^2)} \]

Find \( f_y = e^{x(1+y^2)} \frac{\partial}{\partial y} \left[ x(1+y^2) \right] \)

\[ = e^{x(1+y^2)} \frac{\partial}{\partial y} \left[ x + xy^2 \right] \]

\[ = e^{x(1+y^2)} \left[ 0 + 2xy \right] \]

\[ = 2xye^{x(1+y^2)} \]

**Interpretation**

\( Z = f(x, y) \)

- \( z_x \) is the rate of change of \( Z \) as \( x \) varies with \( y \) fixed
- \( z_y \) is the rate of change of \( Z \) as \( y \) varies with \( x \) fixed
citrus
\downarrow \quad \nabla \quad \text{gunpowder}
\uparrow
F(c, \theta)

\frac{\partial F}{\partial c} \quad \text{marginal rate of change of fighting ability as barrels of citrus vary}

\frac{\partial F}{\partial \theta}

Some economics

Cost of making an item = \( C(x, y) \)

\( c_x \) \quad \text{marginal cost of a more x-input}

\( c_y \) \quad \text{" same " y-input}
Productivity \((P)\) depends at least on labor \((L)\) and capital \((K)\):

\[
P(L, K) = p_0 L^\alpha K^\beta
\]

\(\alpha = \) "alpha" \(\geq 0\)
\(\beta = \) "beta" \(\geq 0\)
\(\alpha + \beta = 1\) "constant returns to scale"
\(\alpha + \beta < 1\) "decreasing returns to scale"
\(\alpha + \beta > 1\) "increasing returns to scale"

Hold \(K\) constant

\[P(L, K) = p_0 L^{0.3} K^{0.7}\]
\[
\frac{\partial}{\partial L} \phi = \psi_0 K^{0.6} \frac{\partial}{\partial L} \left[ -L^{0.3} \right] = \psi_0 K^{0.6} 0.3 L^{0.3-1} = \psi_0 K^{0.6} 0.3 L^{0.3} - 0.7
\]
\[
\frac{\partial p}{\partial L} = \psi_0 K^{0.6} \frac{0.3}{L^{0.7}}
\]
one more interpretation from AUSMIRA

\[ f(x) \quad \frac{df}{dx} \]

"The effect of increasing \( x \) by 1 unit is to change the value of the function from \( f(x) \) to \( f(x) + \frac{df}{dx} \)"

\[ Ay = \frac{df}{dx} \Delta x \]

\( P(L, K) \propto \text{current productivity} \)

so if we increase labor by an amount \( \Delta L \)

\[ P(L, K) \rightarrow P(L, K) + \frac{\partial P}{\partial L} \Delta L \]

\[ \frac{1}{\Delta P} \]
Demand for item type A: $q_A = h(f(p_A, p_B))$

4 partial derivatives "swell"

Demand for item type B: $q_B = h(f(p_A, p_B))$

$p_A =$ unit price of item type A

"Economic axioms":

\[ \frac{\partial q_A}{\partial p_A} < 0 \]

\[ \frac{\partial q_B}{\partial p_B} < 0 \]

If

\[ \frac{\partial q_A}{\partial p_B} > 0 \text{ and } \frac{\partial q_B}{\partial p_A} > 0 \]

"The demand for item A increases as the price of item B goes up."

A & B are said to be competitive products or substitutes.
If \( \frac{\partial q_A}{\partial p_B} < 0 \) and \( \frac{\partial q_B}{\partial p_A} < 0 \)

Thus A and B are complementary

What if \( \frac{\partial q_A}{\partial p_A} = 0 \) essential item

\( \frac{\partial q_A}{\partial p_B} = 0 \) A and B are not related in consumer behavior
§ 17.4 Higher order partial derivatives

$f_{xx} = \frac{\partial^2 f}{\partial x^2}$

\[
\left( \lim_{h \to 0} \frac{f_{x}(x+h,y) - f_{x}(x,y)}{h} \right)
\]

$f_{yy} = \frac{\partial^2 f}{\partial y^2}$

\[
\left( \lim_{h \to 0} \frac{f_{y}(x,y+h) - f_{y}(x,y)}{h} \right)
\]

$f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$

$f_{yx} = \frac{\partial^2 f}{\partial y \partial x}$
It is true that
\[ f_{xy} = f_{yx} \]

\[ f(x, y) = x^4/y + y^2x = x^{y-1} + y^2x \]

\[ f_x = 4x^3y^{-1} + y^2 \]
\[ f_{xx} = 12x^2y^{-1} + 0 \]
\[ f_y = -x^4y^{-2} + 2yx \]
\[ f_{yy} = 2x^4y^{-3} + 2x \]