\[ \int f(x) \, dx \text{ means find the function } \]
\[ \text{whose derivative is } f(x) \]

\[ \int g(x^3) \, dx = \frac{1}{3} x^3 + C \]

\[ \int e^{-s} \, ds = -e^{-s} + C \]

Since
\[ \frac{d}{ds} \left[ e^{-s} \right] = e^{-s} \frac{d}{ds} (-s) = e^{-s} \]

\[ \int \frac{x^3 + x}{x^2} \, dx = \int \left[ \frac{x^3}{x^2} + \frac{x}{x^2} \right] \, dx \]
\[ = \int x \, dx + \int \frac{1}{x} \, dx \]
\[ = \frac{x^2}{2} + \ln(x) + C \]
\[ \int \frac{1}{\sqrt{x}} \frac{1}{(\sqrt{x}-2)^3} \, dx \]

\[ = \int \frac{1}{\sqrt{x}} (\sqrt{x}-2)^{-3} \, dx \]

Try \( u = \sqrt{x} - 2 \)

\[ u'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2 \sqrt{x}} \]

\[ du = u'(x) \, dx = \frac{1}{2 \sqrt{x}} \, dx \]

\[ \frac{1}{2 \sqrt{x}} \, dx = 2 \, du \]

So

\[ \int \frac{1}{\sqrt{x}} (\sqrt{x} - 2)^{-3} \, dx = \int 2u^{-3} \, du \]

\[ = 2 \left[ -\frac{1}{2} u^{-2} + C \right] = -\left( (\sqrt{x}-2)^{-2} + 2 \cdot C \right) \]
\[ \int \frac{1}{x \ln x} \, dx \]

Let's try \( u = \ln x \)

\[ u'(x) = \frac{1}{x} \]

\[ du = u'(x) \, dx = \frac{dx}{x} \]

So

\[ \int \frac{1}{x \ln x} \, dx = \int \frac{1}{\ln x} \, dx = \int \frac{1}{u} \, du \]

\[ = \ln |\ln(x)| + C \]

\( \text{Absolute values used so that we only take log of positive numbers} \)
\[ \int k^u \, du = \frac{u^{k^1}}{ln(k)} + C \] [Power Rule]

\[ \int k^u \, du \]

We "recall" \( k = e \)

We do? Since \( e^x \) and \( \ln(x) \) are inverse functions and

\[ \ln(k^u) = u \ln(k) \]

So

\[ \int k^u \, du = \int e^{u \ln(k)} \, du \]

\[ = \frac{e^{u \ln(k)}}{\ln(k)} + C \]

\[ = \frac{k^u}{\ln(k)} + C \]
Over the years people have made tables of many, many indefinite integrals.

Appendix B

These tables do not replace thinking.

\[ \int a + bu \]
\[ \sqrt{a + bu} \]
\[ \sqrt{a^2 - u^2} \]
\[ \sqrt{u^2 + a^2} \]

Exponential

Miscellaneous
\[ \int \frac{x}{(2+3x)^2} \, dx \]

From the integral table (#7)

\[ \int \frac{udu}{(at+bu)^2} = \frac{1}{b^2} \left[ \ln |at+bu| + \frac{a}{at+bu} \right] + C \]

\[ a = 2 \]
\[ b = 3 \]

So,

\[ \int \frac{x}{(2+3x)^2} \, dx = \frac{1}{9} \left[ \ln |2+3x| + \frac{2}{2+3x} \right] + C \]
\[ \int x^2 \sqrt{x^2-1} \, dx \]

\[ \#24 \text{ from the table} \]

\[ \int \mu^2 \sqrt{\mu^2-a^2} \, d\mu = \frac{\mu}{8} \left( 2\mu^2-a^2 \right) \sqrt{\mu^2-a^2} \]

\[ -\frac{a^4}{8} \ln \left| \mu + \sqrt{\mu^2-a^2} \right| + C \]

\[ \mu = 1 \]

So

\[ \int x^2 \sqrt{x^2-1} \, dx = \frac{x}{8} \left( 2x^2-1 \right) \sqrt{x^2-1} \]

\[ -\frac{1}{8} \ln \left| x + \sqrt{x^2-1} \right| + C \]
Problem 4 A consumer has allotted $I$ dollars a day to pay for coffee (mainly lattes), $c$, and power bars $p$. Lattes cost $3.00 each and power bars $2.00 each. His daily utility function for these is

$$U(c,p) = 2ln(c + 1) + 4ln(p + 1)$$

A) (2 pts) What is the relationship between his allocation and his consumption of power bars and coffee??

B) (2 pts) Suppose that he decides to limit himself to one power bar a day. Sketch $U(c, 1)$.

$$M(c, 1) = 2ln(c + 1) + 4ln(2)$$

C) (2 pts) Interpret your sketch from part B.

D) (2 pts) What are the rates of consumption of coffee and power bars that maximize his utility?
(E) (2 pts) Suppose that he decides to increase his allocation for these items by $2 a day. Approximately how much will his optimal utility increase?

Increase in his optimal utility is the differential

\[
\frac{\Delta \hat{u}^*(t)}{\Delta I} \quad \Delta I = \Delta \hat{u}^*
\]

from the previous part of the problem

\[\Delta I = 2\]

We learned from the envelope theorem that

\[\lambda = \frac{\partial \hat{u}^*(t)}{\partial I}\]

\[\Delta \hat{u}^* = 2 \lambda\]

what you that is
If \( f(x,y) = 3^{xy} \) what is \( f_y \)?

It may be helpful to recall that \( 3^z = e^{\ln(3^z)} \)

Follow the hint:

\[
3^{xy} = e^{\ln(3^{xy})}
\]

\[
3^{xy} = e^{xy \ln(3)} = e^{xy \ln(3)} = e^{xy \ln(3)} = e^{xy \ln(3)}
\]

\[
\frac{d}{dy} \left[ e^{xy \ln(3)} \right] = \frac{d}{dy} \left[ e^{xy \ln(3)} \right] = \left[ \frac{d}{dy} e^{ky} \right] = ke^{ky}
\]

\[
= x \ln(3) e^{xy \ln(3)} = x \ln(3) e^{xy \ln(3)} = x \ln(3) e^{xy \ln(3)} = x \ln(3) \cdot 3^{xy}
\]
The sole producer of a product has determined the marginal revenue function...

\[ \frac{dr}{dq} = 100 - 3q^2 \]

Determine the pt. elasticity of demand for the product when \( q = 5 \). (Hint: 1st find demand function)

**To Find Point elasticity of demand**

**We know:** Marginal revenue

\[ r(q) = q \cdot p(q) \]

Price as a function of demand

Point elasticity of demand

\[ \eta = \frac{q \, \frac{dp}{dq}}{p(q)} \]

"eta"

"price-uh"
First we find $r$:

$$\frac{df}{dq} = 150 - 3q^2$$

$$r(q) = 100q - q^3 + C$$

Since $f(0) = 0$, we have $C = 0$.

So, $r(q) = 100q - q^3 = qf(q)$.

Thus:

$$f(q) = 100 - q^2$$

$$\frac{df}{dq} = -2q$$

$$f/q = \frac{100}{q} - q$$

And these are all the pieces we need...