Faux Final Exam

2-8 December 2009

Instructions

• Please try to mimic exam conditions when you do this (see the front page of your other exams to know what this means).

• I have instructed the TAs and tutors not to do the problems for you. If you have done a problem and want them to check it, fine; if you have started and problem and got stuck, they will show you how to get unstuck and you can then finish it; if you cannot see how to start a problem, they will show you.

• You must bring your student ID to the final exam, to show to us when you turn in the exam.

NAME: ________________________________

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Problem 1

A) (5 points) In class, we discussed the formula for the Taylor polynomial for $f(x)$ around the point $a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Write the first four terms of the Taylor polynomial for $f(x) = e^{-x}$ around $a = 0$. 
B) (5 points) The demand function for a certain item is

\[ p(q) = \frac{3}{q^3 + 1} \]

What is the marginal revenue function when \( q = 2 \)?
Problem 2 The demand equation for a firm’s product is given by

\[ q^4 + 2q^2 \cdot p = 21 \]

A) (4 points) Use implicit differentiation to find \( \frac{dq}{dp} \) and the point elasticity of demand.

B) (4 points) Use your answer from part A) to find the formula for marginal revenue. Hint: Recall that \( \frac{dp}{dq} = \frac{1}{\frac{dq}{dp}} \).
C) (2 points) Compute the point elasticity of demand and marginal revenue when $q = 1$. 


Problem 3.
A) (5 points) Sketch the function, for $x \geq 0$,

$$f(x) = 6xe^{-x}$$

Be sure to justify your choice of numerical values, limits, and critical points.
B) (5 points) Sketch the function

\[ h(y) = \frac{y^2}{y^2 + 3} \]

Be sure to justify your choice of numerical values, limits, and critical points.
Problem 4

A (5 points) Find and classify the relative extrema of

\[ f(x) = x^3 - 9x + 12 \]
B (5 points) If \( r(q) = 2r^3 - 18r - 39 \) is a total-revenue function, determine the intervals on which the marginal revenue is increasing. Hint: Look at your work for part A before starting.
Problem 5 The Royal Navy is provisioning a ship to hunt down, capture, and hang Captain Peter Blood. They have 200 pounds left to use for outfitting the ship. They have to buy citrus, $C$ at 5 pounds a barrel and gunpowder, $G$ at 10 pounds a barrel. They know that the fighting capacity of the ship, $F$ is given by $F = 6 \cdot C^{1/2} \cdot G$.

A) (3 points) What is the relationship between the 200 pounds, the barrels of citrus, and barrels of gunpowder that could be purchased?

B) (7 points) What is the optimal number of barrels of citrus to purchase in order to maximize the fighting capacity of the ship with this purchase? Be certain to justify your answer.
Problem 5 The demand equation for a profit-maximizing firm is given by

\[ p(q) = 100 - 4q \]

and the cost of producing \( q \) items is

\[ c(q) = 1 + 2q \]  \hspace{1cm} (1)

A) (3 points) Sketch the associated revenue function.
B) (8 points) Find the value of production that maximizes profit; be sure to justify your assertion that this is a maximum.
Extra Credit (5 points)

What is your function and why? Be sure to explain in readable and clear sentences.