Review Questions 2
Indefinite Integrals and Applications

1. Compute the following integrals

a. \[ \int \frac{3x \, dx}{\sqrt{x^2 + 1}} = \]
b. \[ \int (x^2 + 2x)(x^3 + 3x^2 - 1)^3 \, dx = \]
c. \[ \int \frac{dx}{x \ln x} = \]
d. \[ \int (x^2 + x)(x^3 + x^2 - 1)^2 \, dx = \]
e. \[ \int \frac{3x \cdot e^{\sqrt{x^2 + 1}} \, dx}{\sqrt{x^2 + 1}} = \]
f. \[ \int 1000e^{-0.05t} \, dt = \]
g. \[ \int 7 \cdot 5^t \, dt = \]
h. \[ \int \frac{2 + x}{2x + 1} \, dx = \]

2. A firm’s marginal cost function is \( \frac{dc}{dq} = 2(5q + 100)^{1/2} \), and their fixed cost is $10000. Find the firm’s cost function.

3. A firm’s marginal revenue and marginal cost functions are

\[ \frac{dr}{dq} = 200 - (2q + 8)^{2/3} \quad \text{and} \quad \frac{dc}{dq} = 0.2q + 65, \]
respectively. How will the firm’s profit change if output is increased from \( q = 100 \) to \( q = 200 \)?

4. A firm’s marginal revenue function is given by \( \frac{dr}{dq} = 50 - \frac{(\ln(q + 1) + 1)^5}{q + 1} \). Find the firm’s revenue function.

5. Suppose that a small nation’s marginal propensity to consume is given by

\[ \frac{dC}{dY} = \frac{63Y^2 + 70Y - 450}{(9Y + 5)^2}, \]
where \( Y \) is income and \( C \) is consumption, both measured in billions of dollars.

a. Compute \( \lim_{Y \to \infty} \frac{dC}{dY} \), and interpret your result in economic terms.

b. Find the function \( C(Y) \), given that \( C(5) = 4.5 \).

c. Compute \( \lim_{Y \to \infty} \frac{C}{Y} \), and interpret your result in economic terms. Compare this to your answer in part a. What does this mean?