EXAM 1
19 October 2009

Instructions

- Please turn your OFF your cell phone, iPod, etc..
- There are 4 questions worth a total of 40 points. 100%=40 points.
- Please write neatly and show all your work.
- No notes or books.
- NO calculators, NO PDAs, etc.
- Take your time, and check your answers.

Use the Method of Thinking!!!

NAME: ANSWER KEY

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Problem 1

A (1 point) What is $\frac{\infty}{\infty}$? \textcolor{red}{\textit{undefined}} \hspace{1cm} (1 pt)

B (4 points) Write the equation of the line that passes through $(-2, -2)$ and is perpendicular to the line $y = 4x - 7$

Slope of line: $-\frac{1}{4}$ \hspace{1cm} (1 pt)

Eqn $\frac{y - (-2)}{x - (-2)} = -\frac{1}{4}$ \hspace{1cm} (3 pts) or $\frac{y + 2}{x + 2} = -\frac{1}{4}$ \hspace{1cm} (3 pts)

C (5 points) The slope of the line tangent to a curve $f(x)$ is given by $x^2 - 3x + 2$. For what values of $x$ is the tangent line horizontal?

$f'(x) = x^2 - 3x + 2$ \hspace{1cm} (1 pt)

Tangent line horizontal means $f'(x) = 0$ \hspace{1cm} (2 pts)

$(x^2 - 3x + 2) = 0$

$(x - 2)(x - 1) = 0$

$x = 2, 1$ \hspace{1cm} (2 pts)
Problem 2
A (4 points) Solve $\log(5x + 10) = 0$ for $x$.

\[
\log(5x+10) = 0 \quad \text{means} \quad 5x+10 = 1 \quad (2 \text{ pts})
\]

\[
5x = -9 \quad (2 \text{ pts})
\]

\[
x = -\frac{9}{5} \quad (2 \text{ pts})
\]

B (2 points) If $\ln(y) = -\frac{1}{2}(x-7)^2$, what is $y$?

\[
\ln(y) \quad \text{means base e} \quad (1 \text{ pt})
\]

\[
y = e^{-\frac{1}{2}(x-7)^2} \quad (1 \text{ pt})
\]

C (4 points) Suppose that $\log_b(m) = z$ and that $y = 4m^3$. What is $\log_b(y)$, expressed in terms of $z$? Do not answer $\log_b(4m^3)$.

\[
\log_b(m) = z \quad \text{means} \quad b^z = m \quad (1 \text{ pt})
\]

\[
y = 4m^3 = 4(b^z)^3 = 4b^{3z} \quad (1 \text{ pt})
\]

\[
\log_b(y) = \log_b(4b^{3z}) \quad (1 \text{ pt})
\]

\[
= \log_b(4) + 3z \quad (1 \text{ pt})
\]
Problem 3
A (1 point) What is \( \lim_{x \to 0} (2x^2 + 5x + 1)? \)

\[
\text{As } x \to 0 \quad 2x^2 + 5x \downarrow 0
\]

so \( \lim = 1 \) \hspace{1cm} (1 pt)

B (2 points) What is \( \lim_{x \to 3} \frac{x-3}{6-2x} \)?

\[
\lim_{x \to 3} \frac{x-3}{6-2x} = \lim_{x \to 3} \frac{(x-3)}{2(3-x)} = \lim_{x \to 3} \frac{-1}{2} = -\frac{1}{2} \quad (1 \text{ pt})
\]

C (3 points) What is \( \lim_{x \to \infty} \frac{2x^2 + 8}{x^2} \)?

\[
\lim_{x \to \infty} \frac{2x^2 + 8}{x^2} = \lim_{x \to \infty} \frac{2x^2 + 8}{x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \to \infty} \frac{2 + \frac{8}{x^2}}{1} = 2 \quad (1 \text{ pt})
\]

D (4 points) For what values of \( a \) and \( n \) is \( \lim_{x \to \infty} \frac{5a^n + 7x}{ax^3 + 12} = \frac{1}{2} \)?

\[
\text{n must match highest power in denominator, thus}
\]

\[
\lim_{x \to \infty} \frac{5x^3 + 7x}{ax^3 + 12} = \lim_{x \to \infty} \frac{(5x^3 + 7x)}{ax^3 + 12} = \lim_{x \to \infty} \frac{5 + \frac{7x}{x^3}}{a + \frac{12}{x^3}} = \frac{5}{a} \quad (1 \text{ pt})
\]

Thus \( \frac{5}{a} = \frac{1}{2} \Rightarrow a = 10 \) \hspace{1cm} (1 pt)
Problem 4
A (2 points) Find the values of $x$ for which $x^2 - 3x + 2$ is positive, zero, or negative.

\[ x^2 - 3x + 2 = (x-1)(x-2) \]

is 0 at $x=1, 2$ (1 pt)

\[ (-)(-)=+ \quad (+)(-)=- \quad (+)(+)=+ \] (1 pt)

\[ 0 \quad 1 \quad 2 \]

B (3 points) What is $\frac{d}{dx}(7x^2 + 2x + 9)$?

\[ \frac{d}{dx}(7x^2 + 2x + 9) = 7 \cdot 2x + 2 \]

\[ = 14x + 2 \] (either 3 pts)

C (5 points) Compute the derivative of $x^2 - 1$, directly from the definition.

\[ \frac{d}{dx}(x^2 - 1) = \lim_{h \to 0} \frac{[(x+h)^2 - 1] - [x^2 - 1]}{h} \] (2 pt)

\[ = \lim_{h \to 0} \frac{[x^2 + 2xh + h^2 - 1] - [x^2 - 1]}{h} \] (1 pt)

\[ = \lim_{h \to 0} \frac{2xh + h^2}{h} \] (1 pt)

\[ = \lim_{h \to 0} h \left[ \frac{2x+h}{h} \right] = \lim_{h \to 0} 2x + h = 2x \] (1 pt)