29 Sept 09

HW #1 DUE NO LATER THAN 8AM MON OCT 5

\[ R(q) = p(q) \cdot q = q \cdot (a - bq) \]
§ 3.4

\[ l_1 : a_1 x + b_1 y = c_1 \quad a, b, c \text{ real numbers} \]

\[ l_2 : a_2 x + b_2 y = c_2 \]

2 equations for 2 unknowns

\[ \begin{align*}
  y & \quad l_1 \\
  l_2 & \quad x \\
\end{align*} \]

One solution

\[ \begin{align*}
  y & \quad l_1 \\
  l_2 & \quad x \\
\end{align*} \]

No solution

\[ \begin{align*}
  y & \quad l_1, l_2 \\
  x & \quad l_1, l_2 \\
\end{align*} \]

Infinitely many solutions
Finding $x$ & $y$

1. By adding/subtracting to get rid of $x$ or $y$ \[ \frac{1}{2} \text{ eqn in } y \text{ unknown} \]

2. Find $y$ in terms of $x$ from line $L_1$, say substitute into line $L_2$
\[ a_1x + b_1y = c_1 \Rightarrow y = \frac{1}{b_1} \left[ c_1 - a_1x \right] \]

Q. When are there infinitely many solutions?
A. When $L_1 = \text{multiple of } L_2$
$F(B) = H(B)$

$P(B) = rB(1 - \frac{B}{K})$ in $\frac{B}{K}$

Annual production of Sandeels when the Sandeel biomass $L$ tons is $B$

Fishing effort $F(B) = cEB$

Harvest $= H(B) = cEB$

Example $x^2 - 5x + y = 6$

$x + y = 1$ substitute $y = 1 - x$
§ 3.6 - OTHER APPLICATIONS

Your job ⇒ read this section

Are these always linear? — No.

Neal Stephenson
\[ p = \frac{8500}{q} \]

\[ p = \frac{q}{40} + 10 \]

\[ \frac{8000}{q} = \frac{q}{40} + 10 \]

\[ 8500 = \frac{q^2}{40} + 10q \]

\[ \text{READ PCS 39-42} \]
3.4.1 Review: Exponents and Logs.

\[ f(x) = b^x \]

Exponential function with base \( b > 0 \)

Rules for exponents

\[ a^m \cdot a^n = a^{m+n} \]

\[ \frac{a^m}{a^n} = a^{m-n} \]

\[ (a^m)^n = a^{mn} \]

\[ \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \]

\[ (a/b)^n = a/n / b^n \]

\[ a^0 = 1 \]

\[ a^1 = a \]

\[ This \ is \ \underbrace{a \cdot a \cdot a \cdots a}_{m \text{ times}} \]

\[ \text{"Eh to the En times} \]

\[ \text{Bee to the En"} \]
\[ f(x) = 2^x \]

\[ f(x) = 2^{-x} = \frac{1}{2^x} \]
Application: Interest Accumulation

P = principle invested

r = rate of interest  [annual]

A(n) = Accumulated value a years in the future

A(n) = P \cdot (1+r)^n

To begin suppose \( r = 1 \)

After one year

\[ A = P \cdot (1+1)^{\frac{1}{1}} = P \cdot 2 \]

After 2 6-month periods

\[ A = P \cdot (1+\frac{1}{2})(1+\frac{1}{2}) = P \cdot (1.5)^2 \]

\[ \text{value after 6 months} \]

After 3 quarter-periods

\[ A = P \cdot (1+\frac{1}{3})(1+\frac{1}{3})(1+\frac{1}{3}) \]

\[ \text{after first trimester} = P \cdot (1.333)^3 \]

\[ = P \cdot 2.37 \]
Many periods \[ A = P \left( 1 + \frac{1}{n} \right)^n = P \left( \frac{n+1}{n} \right)^n \]

As \( n \) gets larger \& larger

\[ \left( 1 + \frac{1}{n} \right)^n = \left( \frac{n+1}{n} \right)^n \]

gets closer and closer to a \underline{special} number

\[ e = 2.71828... \]

\[ \uparrow \]

goes on forever

\[ \underline{Napier} \]

\[ f(x) = e^x \quad \text{a very special function} \]
4.2 Logarithmic Functions

\[ y = b^x \]

\[ \Leftrightarrow \]

\[ x = \log_b y \]

`x` is the number that \( b \) is raised to in order to obtain \( y \).