7 Oct 09

Announcements

• Office hours today, M, W next week and M 19 Oct in BE 160

MSI
• Drop-in tutoring Thurs 3-6 pm
  E2 215

• -6/18 is a correct but unesthetic answer

• MSI sections start today
The slope of the line tangent to $y = f(x)$ at $x = a$ is the limit of the slope of the lines joining $(a, f(a))$ and an adjacent points as the two points get closer and closer together.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a}$$

"Eh" prime of Eh
One more limit
\[
\lim_{h \to 0} \frac{\left( \frac{1}{h} + 2h \right) h^2}{\left( \frac{1}{h^2} + 7 \right) h^2} = \frac{h + 2h^3}{1 + 7h^2} \to 0
\]

§10.3 A function \( f(x) \) is continuous at \( x = a \) if

1. \( f(a) \) exists
2. \( \lim_{x \to a} f(x) \) exists
3. \( \lim_{x \to a} f(x) = f(a) \)

Polynomial functions (powers of \( x \)) are always continuous
A function is discontinuous

1) $f(x)$ has no limit as $x \to a$

2) $f(a)$ is limit as $x \to a$

3) $f(x)$ does not exist when $x = a$

Postage stamp function (USPS function)

Discontinuous because limit as $x \to 1, 2, 3$ etc does not exist
Functions can be discontinuous only at some points.

\[ f(x) = \frac{x^2 + 1}{x^2 - 1} = \frac{x^2 + 1}{(x+1)(x-1)} \]

Is not defined at \( x = 1 \) or \( x = -1 \) and thus is not continuous at those points.
Inequalities or Sign Mapping

\[ f(x) = rx(1 - \frac{x}{K}) \]

\((-)(+) = - \quad (+)(+) = + \quad (+)(-) \rightarrow -\]

\[ f(x) < 0 \quad f(x) > 0 \quad K \quad f(x) < 0 \]

"Not everything in life is a parabola"
What is the sign mapping for

\[ f(x) = x(x^2 - 3x - 10) \]

\[ = x(x-5)(x+2) \]

\[ (-)(-)(-) = - \quad (+)(-)(+) \quad (+)(+)(+) = + \]

\[ f(x) < 0 \quad f(x) > 0 \quad f(x) < 0 \quad f(x) > 0 \]

\[ -2 \quad 0 \quad 5 \quad x \]

\[ (-)(-)(+) \]
§ 11.1 The Derivative

The slope of the line tangent to \( y = f(x) \) at \( x = a \) is called the derivative of \( y \) with respect to \( x \) evaluated at \( x = a \).

We write

\[
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
\]

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]
Other notation
\[
\frac{df}{dx} \quad \frac{d}{dx} (f(x))
\]

"Dee Ex"  "Dee by Dee Ex of Eff of Ex"

\[
\frac{dy}{dx} \quad y'(x) \quad D_x y
\]

"Dee sub Ex of why"

\[
D_x (f(x))
\]

All mean
\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{Definition of the derivative}
\]

Your book
\[
\lim_{z \to x} \frac{f(z) - f(x)}{z-x}
\]
Example \( f(x) = 7x \)

\[
\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{7x+7h - 7x}{h}
\]

\[
= \lim_{h \to 0} \frac{7h}{h} = \lim_{h \to 0} 7 = 7
\]

When!

Example \( f(x) = x^2 \)

\[
\frac{df}{dx} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}
\]

\[
= \lim_{h \to 0} \frac{(x+h)(x+h) - x^2}{h}
\]

Come back Friday to find out what this derivative is!