A = P (1.05)^t

How long until \( A = 2P \)
\[
2P = P (1.05)^t
\]
\[
t = \frac{\log_2 2}{\log_2 (1.05)}
\]

\[
2 = (1.05)^t
\]
\[
t = \frac{\log_{10} 2}{\log_{10} 1.05}
\]

\[
t = \left( \frac{\log_{10} 2}{\log_{10} 1.05} \right)
\]
Logarithmic Sweetness

\[ \log_b m = z \]

Means

\[ b^z = m \]

Take \( \log_a(\cdot) \) of both sides

\[ \log_a b^z = \log_a m \]

\[ z \log_a b = \log_a m \]

\[ (\log_b m)(\log_a b) = \log_a m \]

\[ \log_b m = \frac{(\log_a m)}{(\log_a b)} \]

\[ \rightarrow \text{ sweet} \]
Big idea from last lecture: tangent line

The slope of the line tangent to $y = f(x)$ at the point $x = a$ is the limit of the slope of the line between $(a, f(a))$ and $(x, f(x))$ as $x \to a$.

This slope is $\frac{f(x) - f(a)}{x - a}$. 
We say that the limit of \( f(x) \) as \( x \to a \) is \( m \), written as

\[
\lim_{x \to a} f(x) = m
\]

if as \( x \) gets closer and closer to \( a \), \( f(x) \) gets closer and closer to \( m \).

\[
\lim_{x \to 2} (x+3) = 5
\]

\[
\lim_{x \to 1} \frac{x^3-1}{x-1} = 3
\]

Some limits may not exist.

\[
\lim_{x \to 0} \frac{1}{x}
\]
Discontinuous function (for any $n$ no limit)

Properties of limits $a, b, c$ numbers

\[
\lim_{x \to a} c = c
\]

\[
\lim_{x \to a} x^n = a^n \quad \text{for } n > 0
\]

If $f$ and $g$

\[
\lim_{x \to a} f(x) = L_f
\]

\[
\lim_{x \to a} g(x) = L_g
\]
\[
\lim_{x \to a} \left[ f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)
\]

plus or minus, but not both

\[
\lim_{x \to a} \left[ f(x) \cdot g(x) \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)
\]

\[
\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)
\]

Remember: Limits can be tricky things

Example: \[ f(x) = \frac{x^2 - 1}{x+1} \] is not defined at \( x = -1 \)

but
\[
\lim_{x \to -1} \frac{x^2 - 1}{x+1} = \lim_{x \to -1} \frac{(x+1)(x-1)}{x+1}
\]

\[
= \lim_{x \to -1} (x-1) = -2
\]
\[
\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e
\]

If let \( x = \frac{1}{n} \)
\[
\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e
\]

§ 10.2 Some More About Limits

We can talk about left-hand and right-hand limits
\[\lim_{x \to a^-} f(x) \quad \lim_{x \to a^+} f(x)\]

The limit of \( f(x) \) as \( x \to a \) exists only if
\[\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)\]
We also have to be smart about things.

\[ f(x) = \sqrt{x-4} \]

\[ f(4) \]

\[ \lim_{x \to 4} \sqrt{x-4} = 0 \]

Infinite limits

\[ \lim_{x \to 0} \frac{1}{x} \, \text{is not defined} \]

\[ \lim_{x \to \infty} \frac{1}{x^2} \] is getting closer and closer to $0$, something mystical, the limit is not defined.
Limits as $x \to \infty$

\[
\lim_{x \to \infty} \frac{1}{x} = 0
\]

\[
\lim_{x \to \infty} \frac{ax}{1+bx}
\]

We note as $x \to \infty$ $bx+1$ is closer and closer to $bx$

Let's divide top and bottom by $x$

\[
\lim_{x \to \infty} \frac{ax}{1+bx} = \lim_{x \to \infty} \frac{ax}{(1+bx)\frac{1}{x}}
\]

\[
\lim_{x \to \infty} \frac{a}{(\frac{1}{x}+b)} = \frac{a}{b}
\]

\[
\lim_{x \to \infty} \frac{ax}{1+bx} = \frac{a}{b}
\]
\[
\lim_{x \to 0} \frac{9x}{1+6x} = 0
\]

Audience participation problem (APPs)

\[
\lim_{x \to \infty} \frac{x^3 + 2x^2 + 5x}{8x^3 + 4x^2 + 12x}
\]

\[
\lim_{x \to \infty} \frac{(x^3 + 2x^2 + 5x) \frac{1}{x^3}}{(8x^3 + 4x^2 + 12x) \frac{1}{x^3}} = \frac{1}{x^3}
\]

\[
\lim_{x \to \infty} \frac{1 + \frac{2}{x} + \frac{5}{x^2}}{8 + \frac{4}{x} + \frac{12}{x^2}} = \frac{1}{8}
\]
§10.3 Continuity

A function $f(x)$ is continuous at $x = a$ if

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

On the graph:
- Continuous at $x = a$
- Discontinuous because $\lim_{x \to a} f(x) \neq f(a)$
A polynomial function \( f(x) = a_0 + a_1 x + \ldots + a_n x^n \) is always continuous.