Today is the first day of the rest of your life. The rest of your life is word problems!!

Section 13.6 Applied maxima & minima

1. Verbiage describing a relationship between two variables.
2. More verbiage about something that depends on those variables and that must be optimized/minimized/maximized.

How do we proceed
1a. Pick your symbols
1b. Identify the relationship

2a. Mathematically characterize the quantity that depends on these variables.
2b. Express one variable in terms of the other.
2c. Rewrite the result from part 2a in terms of just one variable.

3a. Take the first and derivatives, set the the first derivative equal to 0 and use the second derivative to confirm a max/min as required by the verbiage.
Problems 13.6

In this set of problems, unless otherwise specified, p is price per unit (in dollars) and q is output per unit of time. Fixed costs refer to costs that remain constant at all levels of production during a given time period. (An example is rent.)

1a) Find two numbers whose sum is 82 and whose product is as big as possible.

1b) \( x + y = 82 \)

2a) Product \( P = x \cdot y \)

2b) \( x = 82 - y \)

2c) \( P = x \cdot y = (82 - y) \cdot y = 82y - y^2 \)

3a) \( \frac{\partial P}{\partial y} = 82 - 2y \) \( \frac{\partial^2 P}{\partial y^2} = -2 \quad \text{(max)} \)

\( \frac{\partial P}{\partial y} = 0 \Rightarrow 82 - 2y = 0 \)

\( y = 41 \) \( \{ \text{problem is solved} \} \)

\( x = 41 \)
3. Fencing  A company has set aside $9000 to fence in a rectangular portion of land adjacent to a stream by using the stream for one side of the enclosed area. The cost of the fencing parallel to the stream is $15 per foot installed, and the fencing for the remaining two sides costs $9 per foot installed. Find the dimensions of the maximum enclosed area.

1a) \[ W \text{ ($} \$9 \text{ ft}) \]

\[ L \text{ ($} \$15 \text{ ft}) \]

1b) Cost parallel to the stream: \( 15L \)
Cost perpendicular to the stream: \( 18W \)

\[ 9000 = 15L + 18W \]

2a) Area \( A = L \cdot W \)

2b) \[ 9000 - 18W = 15L \]
3,000 - 6W = 5L
600 - (4/5)W = L

2c) \[ A = L \cdot W = (600 - (4/5)W)W \]
\[ A = 600W - (4/5)W^2 \] (parabola)

3a) \[ \frac{\partial A}{\partial W} = 600 - 2 \cdot (4/5)W \]
\[ \frac{\partial^2 A}{\partial W^2} = -12/5 \] (max)

Set \[ \frac{\partial A}{\partial W} = 0 \Rightarrow 600 = 2 \cdot (4/5)W \]
\[ 300 = (4/5)W \]
\[ 300 \cdot \frac{5}{4} = W \]
\[ 50 \cdot 5 = W = 250 \text{ ft} \]
\[
L = 600 - \left( \frac{6}{5} \right)W
\]

\[
= 600 - \left( \frac{6}{5} \right) \cdot 250
\]

\[
L = 600 - 300 = 300 \text{ ft}
\]

Answer:

\[
\frac{L}{W} = \frac{300 \text{ ft}}{250 \text{ ft}}
\]
Average Cost  A manufacturer finds that the total cost \( c \) of producing a product is given by the cost function

\[
c = 0.05q^2 + 5q + 500
\]

At what level of output will average cost per unit be a minimum?

Given: Total cost \( c(q) = 0.05q^2 + 5q + 500 \)

Average cost

\[
\bar{c}(q) = \frac{c(q)}{q} = 0.05q + 5 + \frac{500}{q}
\]

Kind of like steps 1 & 2

Int step 3

\[
\frac{d}{dq} \bar{c}(q) = 0.05 - \frac{500}{q^2}
\]

\[
\frac{d^2}{dq^2} \bar{c}(q) = \frac{1000}{q^3} \quad (\text{minimum})
\]
\[ \frac{\partial}{\partial q} \bar{c}(q) = 0 \]

\[ .05 = \frac{1}{20} = \frac{500}{q^2} \]

\[ q^2 = 20 \cdot 500 = 10,000 \]

\[ q = \sqrt{10,000} \text{ units} \]
*7. Revenue The demand equation for a monopolist's product is
\[ p = -5q + 30 \]

At what price will revenue be maximized?

\[ r(q) = pq = q(-5q + 30) = -5q^2 + 30q \]

\[ \frac{dr}{dq} = -10q + 30 \]

\[ \frac{d^2r}{dq^2} = -10 \Rightarrow \text{max} \]

Set \[ \frac{dr}{dq} = 0 \Rightarrow -10q^* + 30 = 0 \Rightarrow q^* = 3 \]

Not done yet

The revenue maximizing price is
\[ p^* = -5q^* + 30 = -5(3) + 30 = 15 \]

We are done
What is the purpose of an education?

"The purpose of an education is to expand a mind, not fill it up."

— Adlai E. Stevenson

"Expand a mind" = being able to face new situations, sometimes in a fog of ambiguity, and deal with them.

Tehran 1979

02 1962
Adlai Stevenson

From Wikipedia, the free encyclopedia

This is about the mid-20th-century politician and diplomat who was the Democratic presidential nominee in 1952 and 1956; for other American politicians so named, see Adlai Stevenson (disambiguation).

Adlai Ewing Stevenson II (February 5, 1900 – July 14, 1965) was an American politician, noted for his intellectual demeanor, eloquent oratory, and promotion of liberal causes in the Democratic Party. He served one term as governor of Illinois, and received the Democratic Party's nomination for president in 1952 and 1956; both times he was defeated by Republican Dwight D. Eisenhower. He sought the Democratic presidential nomination for a third time in the election of 1960, but was defeated by Senator John F. Kennedy of Massachusetts. After his election, President Kennedy appointed Stevenson as the Ambassador to the United Nations; he served from 1961 to 1965. He died on 14 July 1965 in London, England after suffering a fatal heart attack at age 65.

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Childhood, education, and early career

Although Stevenson was born in Los Angeles, he was a member of a famous Illinois political family. His grandfather Adlai E. Stevenson I had been Vice President of the United States under President Grover Cleveland from 1889–1897. His father, Lewis Green Stevenson, never held an elected office, but was appointed Secretary of State of Illinois and was considered a strong contender for the Democratic vice-presidential nomination in 1928. A maternal great-grandfather, Jesse W. Fell, had been a close friend and campaign manager for Abraham Lincoln; Stevenson often referred to Fell as his "favorite" ancestor. His mother was Helen Davis Stevenson.

Stevenson was raised in the small city of Bloomington, Illinois; his family was a member of Bloomington's upper class and lived in one of the city's well-to-do neighborhoods. At the age of twelve Stevenson accidentally killed Ruth Merwin, a 16-year-old friend, while demonstrating drill technique with a rifle, inadvertently left loaded, during a party at the Stevenson home. Stevenson was devastated by the accident and rarely referred to it as an adult. However, as the Governor of Illinois he was told about a teenager who had survived an automobile accident while his friend was killed. Stevenson told the teen's father that he should tell his son that "he now has to live for two", which Stevenson's friends took to be a reference to the shooting incident.

Stevenson left Bloomington after his junior year in high school and received his diploma from University High School in Normal, Illinois, Bloomington's "twin city" just to the north. After high school, he attended

http://en.wikipedia.org/wiki/Adlai_Stevenson
Another Message from Bethann Pfluegesen

“When I was taking probability, I got a 38% on the first exam and had to force myself not to throw up while the instructor went over the answers. My Dad said ‘It’s really good that this happened now rather than in your first year of grad school’. I got the second highest grade on the second exam because I was so flipped out that that I worked really hard. I truly did not study properly for the first exam. And in the course I got a solid B and my teacher sent an email at the end saying that he had never seen anybody have as a good a comeback as I did”.
From the syllabus

Grades will be determined as follows

- The average of your 6 highest homework scores: 20%
- Your highest midterm exam: 25%
- Your second highest midterm exam: 15%
- Your lowest midterm exam: 10%
- Your final exam: 30%

Call this Plan A.

For those who were in class the day I took attendance the mean for the three exams is 78.9%

For those who were not in class the day I took attendance the mean for the three exams is 66.2%
Plan B: Grades will be determined as follows

- The average of your 6 highest homework scores: 20%
- Your highest midterm exam: 25%
- Your second highest midterm exam: 15%
- Your lowest midterm exam: 5%
- Your final exam: 35%

For those who were in class the day I took attendance the mean for the three exams is 81.2%

For those who were not in class the day I took attendance the mean for the three exams is 68.4%
The End Game

What I will do:

• I will post a practice final on the course web site by close of business on
  Weds, 2 Dec 09.

• I will use both Plan A and Plan B in computing your final score, and will
  use the larger value when assigning letter grades.

What I would like you to do:

• Work harder in preparation for and take this final more seriously than you
  have ever taken an exam in a mathematics course.

Remember (also from the syllabus)

Letter grades will be determined as follows: Fail: <50%, D: 50-61%, C: 62-73%,
B: 74- 86%; A: >86%.