Office Hours During Finals Week

Monday, Dec 7
9:30-11:30    BE 145
1:00-3:00     BE 160

Tuesday, Dec 8
10:00-12:00   BE 160
1:00-3:00     BE 145

Speak to the MSI tutors directly about their sessions
Faux Final Exam
2-8 December 2009

Instructions

- Please try to mimic exam conditions when you do this (see the front page of your other exams to know what this means).

- I have instructed the TAs and tutors not to do the problems for you. If you have done a problem and want them to check it, fine; if you have started and problem and got stuck, they will show you how to get unstuck and you can then finish it; if you cannot see how to start a problem, they will show you.

- You must bring your student ID to the final exam, to show to us when you turn in the exam.

NAME: ____________________________

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<th>Problem</th>
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Course Slogans

'We can, we will' -- Motto of the US Army 9th Cavalry Regiment (the Buffalo Soldiers)

'Try not. Do, or do not. There is no try' – Jedi Master Yoda

'Simple, but not easy' – Dave Brubeck

What I expect you to know coming into AMS11A

• Operations on real and complex numbers, polynomials and rational expressions;
• Exponents and radicals;
• How to solve linear and quadratic equations;
• How to graph linear and quadratic functions;
• A passing grade in AMS 3, Math 2B or 3, a score of 31 or higher on the placement examination, or approved transfer credit.

Fiat Lux
9. Weight Gain  A group of biologists studied the nutritional effects on rats that were fed a diet containing 10% protein. The protein consisted of yeast and cottonseed flour. By varying the percent p of yeast in the protein mix, the group found that the (average) weight gain (in grams) of a rat over a period of time was

\[ f(p) = 160 - p - \frac{900}{p + 10} \quad 0 \leq p \leq 100 \]

Find (a) the maximum weight gain and (b) the minimum weight gain.

We need to find the values of p that make \( f'(p) \) a max or a min.

\[ f(p) = 160 - p - \frac{900}{p + 10} \]

\[ f'(p) = -1 + \frac{900}{(p+10)^2} = -1 + 900(p+10)^{-2} \]

\[ f''(p) = -1800(p+10)^{-3} \]

We set \( f'(p) = 0 \)

\[ 1 = \frac{900}{(p+10)^2} \]

\[ (p+10)^2 = 900 \]

\[ p = -10 \pm \sqrt{900} \]

\[ p = -10 \pm 30 \]

\[ p = 20, -40 \]

We find the maximum and minimum weight gains at these values of p.
\[
(p+10)^2 = 900
\]
\[
p + 10 = \pm 30
\]
\[
p + 10 = \begin{cases} 
20 \\
-30
\end{cases}
\]
\[
p = \begin{cases} 
20 \\
-40
\end{cases}
\]

\(p = 20\) is a critical point.

\[
f''(20) = -1800 \cdot (20+10)^{-3}
\]
\[
= -\frac{1800}{(30)^3} < 0
\]

\[
\text{At } p = 20\%, \text{ it gives the maximum weight gain.}
\]
The minimum must be at one of the end points

\[ f(p) = 160 - \frac{p - 900}{p + 10} \]

\[ f(0) = 160 - 0 - \frac{900}{10} = 160 - 90 = 70 \]

\[ f(100) = 160 - 100 - \frac{900}{110} \]

\[ = 60 - \frac{90}{11} \]

The minimum weight gsm occurs at \( p = 100 \% \).
11. **Profit**  For a monopolist's product, the demand function is
\[ p = 85 - 0.05q \]
and the cost function is
\[ c = 600 + 35q \]

At what level of output will profit be maximized? At what price does this occur, and what is the profit?

We need to find 3 things:
- **Thing 1:** \( q^* \) (optimizing output)
- **Thing 2:** \( p(q^*) \) (price at \( q^* \))
- **Thing 3:** \( \Pi(q^*) \) (profit at \( q^* \))

**Thing 1**
\[ p = 85 - 0.05q \]
\[ r(q) = p(q) \cdot q = 85q - 0.05q^2 \]
\[ \Pi(q) = r(q) - c(q) \]
\[ = 85q - 0.05q^2 - (600 + 35q) \]
\[ = 85q - 0.05q^2 - 600 - 35q \]
\[ \Pi(q) = 50q - 0.05q^2 - 600 \]

**Take the derivative** \( \Pi'(q) = 50 - 0.1q \)
\[ \Pi'(q^*) = 0 \quad \text{since } q^* \text{ is the optimizing output} \]

\[ 50 - 0.1q^* = 0 \]

\[ q^* = 500 \quad \text{[Thing 1]} \]

**Thing 2**

\[ p(q) = 85 - 0.05q \]

\[ p(q^*) = 85 - (0.05)(500) \]

\[ = 85 - (5 \times 10^{-2})(5 \times 10^2) \]

\[ = 85 - 25 = 60 \quad \text{[Thing 2]} \]
\[ \pi(q) = 50q - 0.05q^2 - 600 \]

\[ \pi(q^*) = 50 \cdot 500 - (5 \times 10^{-2})(500)^2 - 600 \]
\[ = 25 \times 10^3 - (5 \times 10^{-2})(25 \times 10^4) - 600 \]
\[ = 25 \times 10^3 - 125 \times 10^2 - 600 \]
\[ = 25,000 - 1,250 - 600 \]
\[ = 22,150 - 600 \]
\[ = 21,550 \]
17. **Profit** For XYZ Manufacturing Co., total fixed costs are $1200, material and labor costs combined are $2 per unit, and the demand equation is

\[ p = \frac{100}{\sqrt{q}} \]

What level of output will maximize profit? Show that this occurs when marginal revenue is equal to marginal cost. What is the price at profit maximization?

We know how to do this kind of problem well!

We first need to find the cost equation

\[ C(q) = 2q + 1200 \]

\[ \uparrow \quad \uparrow \]

fixed cost

variable cost

\[ (q) = q \cdot p(q) = q \cdot \frac{100}{\sqrt{q}} = q^{4/2} \frac{100}{q^{4/2}} \]

\[ = 100q^{1/2} \]
\[ \Pi(q) = c(q) - c(q) \]
\[ = 100q^{1/2} - (2q + 1200) \]

We set \( \Pi'(q) = 0 \) to find \( q^* \) the optimizing output.

\[ \Pi(q) = 100 \cdot \frac{1}{2} q^{1/2} - 2 \]

\[ \Pi(q) = \frac{50}{\sqrt{q}} - 2 \]

so \( q^* \) satisfies

\[ \sqrt{q^*} \left( \frac{50}{\sqrt{q^*}} = 2 \right) \]

\[ 25 = \frac{50}{2} = \sqrt{q^*} \]

\[ q^* = 625 \]
21. **Container Design**  A container manufacturer is designing a rectangular box, open at the top and with a square base, to have a volume of 32 ft³. If the box is to require the least amount of material, what must be its dimensions? (See Figure 13.69.)

![Figure 13.69](image)

**We know the volume of the box**

\[ V = 32 \text{ ft}^3 = x^2 y \]

**We need to pick \(x, y\) to minimize the amount of material used**  
\(\Rightarrow\) minimize the surface area

\[ S = x^2 + 4 \times x \cdot y \]

- base
- sides
We are in the stratosphere. Summary

\[ \text{minimize } S = x^2 + 4xy \]

with \( 32 = x^2y \)

We can get \( S \) to have just one variable by noting

\[ y = \frac{32}{x^2} \]

then \( S = S(x) = x^2 + 4x \cdot \frac{32}{x^2} \)

\[ S(x) = x^2 + \frac{128}{x} \]

\[ S'(x) = 2x - \frac{128}{x^2} = 2x - 128x^{-2} \]

\[ S''(x) = 2 + 256x^{-3} \]

so the critical point is a minimum.
\[ S'(x) = 2x - \frac{128}{x^2} \]

So the area minimizing side, \( x^* \), satisfies

\[
S'(x^*) = 0
\]

\[
\left( 2x^* - \frac{128}{(x^*)^2} \right) \frac{(x^*)^2}{2} = 0
\]

\[
(x^*)^3 = 64
\]

\[
x^* = (64)^{\frac{1}{3}} = 4 \text{ ft}
\]

We still need to find \( y^* \) from the volume

\[
32 \text{ ft}^3 = (4 \text{ ft})^2 y^*
\]

\[
2 \text{ ft} = y^*
\]