\[ p = \frac{1}{4} q + 6 \]
\[ p = \frac{2240}{q + 12} \]
\[
\frac{1}{4} q + 6 = \frac{2240}{q + 12}
\]
\[ q + 24 = \frac{8960}{q + 12} \]
\[ (q + 24)(q + 12) = 8960 \]
\[ q^2 + 36q + 24 \cdot 12 = 8960 \]
\[ q^2 + 36q + 24 \cdot 12 - 8960 = 0 \]
\[ a \cdot x^2 + b \cdot x + c = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Last time's big new idea

As \( n \) gets bigger and bigger

\[
(1 + \frac{1}{n})^n
\]

gets closer and closer to

\[
e = 2.71828...
\]

We also write

\[
\lim_{n \to \infty} (1 + \frac{1}{n})^n = e
\]

§ 4.2 Logarithmic Functions

\[
10^2 = 100
\]

is the number of times 10 is multiplied by itself to get 100. We write

\[
\log_{10} 100 = 2
\]
\[ 125 = 5^3 \]
\[ \log_5 125 = 3 \]

\[ \log_b x = y \]
\[ \uparrow \]
\[ b^y = x \]

\[ y = b^x \quad b > 1 \]
What about

\[ y = \log_b x \]

\[ b > 1 \]

Since \( y \) is the power that we raise \( b \) to in order to get \( x \)

\((1, b)\) is on this graph since \( b^0 = 1 \)

Inverse functions

\[ \log_b b^x = x \]

\[ b^{\log_b x} = x \]
Base 10 and base e are very important.

\[
\log_{10} \leftrightarrow \log_e \\
\ln \leftrightarrow \log_e
\]

Solve for \(x\):

\[
\log_2 x = 4
\]

Means

\[
2^4 = x \Rightarrow x = 16
\]

\[4.3 \text{ Properties of Logarithms} \]

\[
\log_b (mn) = \log_b (m) + \log_b (n) \\
\log_b \left(\frac{m}{n}\right) = \log_b (m) - \log_b (n) \\
\log_b (m^r) = r \log_b (m)
\]
Simplify

$$\log_b(\frac{1}{x^3}) = \log_b(1) - \log_b(x^3)$$

$$= 0 - \log_b(x^3)$$

$$= -3 \log_b(x)$$

Alternative:

$$\log_b(\frac{1}{x^3}) = \log_b(x^{-3})$$

$$= -3 \log_b(x)$$

§ 4.4 Log & Exponential Eqns

\[ 2 \ln(x+4) = 5 \]

\[ \ln(x+4) = 2.5 \]

\[ \log_e(x+4) = 2.5 \]

\[ x+4 = e^{2.5} \]

\[ x = e^{2.5} - 4 \]
Solve for $x$: $2^{3x} = 7$.

$\log_2 2^{3x} = \log_2 7$

$3x = \log_2 7$

$x = \frac{\log_2 7}{3}$

$17.6^{3x} = 7$

$\log_{10} (17.6^{3x}) = \log_{10} 7$

$3x \cdot \log_{10} (17.6) = \log_{10} 7$

$x = \frac{\log_{10} 7}{3 \cdot \log_{10} (17.6)}$
\[ \lim_{n \to \infty} (1 + \frac{1}{n})^n = e \]

Demand curve

\[ p = 80 - 3q \]

The slope tells us how a change in quantity is related to a change in price.

What about \[ p = \frac{2240}{q+12} \]?
As these two points get closer and closer together, the slope of the line between them is getting closer and closer to the slope of the line tangent to the curve.
We need to think more about limits.

The limit of \( f(x) \) as \( x \) gets closer and to \( a \) is the value that \( f(x) \) gets closer and closer to.

Note: There may not be a limit.

\[
\lim_{x \to 2} (x+3) = 5
\]

Do not plug \( x = a \) into \( f(x) \).

\[
\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3 \text{ (let's make a table)}
\]