<table>
<thead>
<tr>
<th>Question</th>
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<th>4</th>
<th>5</th>
<th>Total</th>
<th>Percent</th>
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<tr>
<td>Score</td>
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<td>Out of</td>
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<td>75</td>
<td>100</td>
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Name ___________________________  AMS 10 Midterm Exam
Section day (Mon Wed or Fri) ____________  Spring Quarter 2009

1. **Complex numbers** (15 points total)
   
   (a) (5 points) Sketch the Argand diagram to show where the complex number $z_0 = -1 + i$ is located. Indicate what $r$ and $\theta$ of the polar form correspond to in your diagram.
   
   ANSWER:

   (b) (5 points) Convert $z_0$ to polar form.

   ANSWER:

   (c) (5 points) Solve $z^4 = z_0$

   ANSWER:
2. **Linear systems** (15 points total)

This question concerns the following linear system:

\[
\begin{align*}
    x_1 + 4x_2 - 4x_3 + 4x_4 &= 5 \\
    2x_1 - x_2 + x_3 - x_4 &= 1 \\
    x_1 + x_2 - x_3 + x_4 &= 2
\end{align*}
\]

(a) (2 points) Write the system as a matrix equation \( Ax = b \) and construct the augmented matrix \( M = [A|b] \) for this system.

ANSWER:

(b) (5 points) Reduce the augmented matrix to **reduced row echelon form**. Circle the **pivots** in the result.

ANSWER:
(c) (2 points) What is the rank of $A$ and the rank of $M$?
ANSWER:

(d) (1 point) How many solutions are there to this system (0,1, or $\infty$)?
ANSWER:

(e) (1 point) Which variables are free parameters (if any)?
ANSWER:

(f) (4 points) Write out the solution $(x_1, x_2, x_3, x_4)$ to the linear system.
ANSWER:
3. **Applications** (15 points total) A company makes three different assortments of nuts - Regular, Superior and Super-Delicious. Each assortment is assembled from a certain ratio of peanuts, walnuts and cashew nuts as shown in the table below, along with the price paid per pound for each of the assortments.

<table>
<thead>
<tr>
<th></th>
<th>Peanuts</th>
<th>Walnuts</th>
<th>Cashews</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>40%</td>
<td>60%</td>
<td></td>
<td>$1.80</td>
</tr>
<tr>
<td>Superior</td>
<td>40%</td>
<td>40%</td>
<td>20%</td>
<td>$4.00</td>
</tr>
<tr>
<td>Super-delicious</td>
<td>20%</td>
<td>50%</td>
<td>30%</td>
<td>$4.40</td>
</tr>
</tbody>
</table>

How much does a pound of each of the different types of nuts cost? (Hints: 1. You can rescale the system to deal with integers instead of fractions, but some weird fractions do come out in the answer anyway. 2. Gaussian elimination and back substitution is less work than full Gauss-Jordan elimination!)

**ANSWER:**
4. **Inverses of matrices** (15 points total) Find the solution of the following linear system by inverting the coefficient matrix.

\[
\begin{align*}
x_1 + 3x_2 + 3x_3 &= 12 \\
x_1 + 4x_2 + 3x_3 &= -10 \\
x_1 + 3x_2 + 4x_3 &= 16 \\
\end{align*}
\]

**ANSWER:**
5. **MATLAB** (15 points total)

Write down the essential Matlab commands you would need to perform the following operations:

(a) (3 points) Find the roots of the following complex equation

\[ 3z^3 - 2iz^2 = 1 + 2i \]

**ANSWER:**

(b) (3 points) Plot the real 2D plane defined by \( z = x - y \) in 3D space on a 11 \( \times \) 11 grid ranging from \(-5 \leq x \leq 5\) and \(-5 \leq y \leq 5\).

**ANSWER:**

(c) (3 points) Find the reduced row echelon form of

\[
A = \begin{pmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
\end{pmatrix}
\]

**ANSWER:**

(d) (3 points) Find the inverse of

\[
B = \begin{pmatrix}
1 & 2 \\
3 & 4 \\
\end{pmatrix}
\]

**ANSWER:**

(e) (3 points) Figure out what the upper and lower triangular matrices are that correspond to the matrix

\[
C = \begin{pmatrix}
1 & 2 & 3 \\
3 & 2 & 1 \\
2 & 1 & 3 \\
\end{pmatrix}
\]

**ANSWER:**