Classical Probability.

Simple events - all equally likely
one of them occurs with probability \( \frac{1}{n} \)

If event A is a set of s simple events

\[ P(A) = \frac{s}{n} \]

Rules of Probability.

\( 0 \leq P(A) \leq 1 \)

event A never occurs

event A always occurs.

event A: complement called "not A"

\[ P(A) \quad \# \quad P(\overline{A}) \quad P(A) + P(\overline{A}) = 1 \]
Flip a coin 3 times
what's probability at least one head.

1 H
2 H
3 H

A = at least one head.

\( \overline{A} = \) zero heads on three coin flips

\( P(\overline{A}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}. \)

\( P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{8} = \frac{7}{8}. \)

Addition Rule

\[ P(A \text{ or } B) \]

Example: roll at least one 6 on two dice.

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & x & x & x & x & x \\
2 & x & & & & \\
3 & x & & & & \\
4 & x & & & & \\
5 & x & & & & \\
6 & x & & & & \\
\end{array}
\]
\[ P(\text{1 on 1st die}) + P(\text{1 on 2nd die}) - P(\text{1 on both}) \]

\[ P(A \cap B) = P(A) + P(B) - P(A \text{ and } B) \]

If \( A \) and \( B \) are **mutually exclusive**, only one of them can occur.

\[ \Rightarrow P(A \text{ and } B) = 0 \]

- \( A \) roll an odd number
- \( B \) roll a six

for mutually exclusive events.

\[ P(A \cap B) = P(A) + P(B) \]
Multiplication Rule.

\[ P(A \text{ and } B) = P(A) \times P(B \mid A) \]
conditional probability.

Prob that \( \text{prob that } B \text{ happens} \)

A happens \( \text{if } A \text{ has already happened.} \)

If \( A \) and \( B \) are independent then the result
of \( A \) does not affect the possibility of
the result of \( B \).

(cg coin tosses; sampling with replacement).

otherwise the events are dependent.

\[ P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}. \]

Told that flights from SJC \( \rightarrow \) LAX leave
on time with prob 0.8
prob that a flight both leaves on time and
arrives on time is 0.72.
If it is known that a flight left on time, what's the probability that it arrives on time?

\[ p(\text{arrives on time} \mid \text{departed on time}) = \frac{p(\text{arrives + departs on time})}{p(\text{departs on time})} = \frac{0.72}{0.8} = 0.9. \]

\[
P(A \text{ and } B) = P(A) \times P(B \mid A)
\]

\[
P(B \text{ and } A) = P(B) \times P(A \mid B)
\]

\[
P(B \mid A) = \frac{P(A \mid B) \cdot P(B)}{P(A)}
\]

Bayes' Theorem.

I have a 2-headed coin + a regular coin. I choose one coin at random + flip it. It comes up heads. What's the probability it was the two-headed coin?
\[
P(2H \mid \text{came up}) = \frac{P(\text{came up} \mid 2H) \times P(2H)}{P(\text{came up} \mid 2H) \times P(2H) + P(\text{came up} \mid \text{regular coin}) \times P(\text{regular coin})}
\]

\[
P(2H) = P(A \mid B) \times P(B).
\]

\[
P(B \mid A) = \frac{P(A \mid B) \times P(B)}{P(A) \times P(B) + P(A \mid \overline{B}) \times P(\overline{B})}
\]

\[
P(2H \mid \text{came up}) = \frac{1 \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{4}} = \frac{2}{3}.
\]
A crime is committed in the Bay Area.
A suspect is arrested and given a DNA test.
Test is positive.
Test is 99.999% accurate.
There is no other evidence than the DNA test.

\[
p(\text{true test} | \text{guilty})
\]
\[
p(\text{guilty} | \text{true test})
\]

Test everyone in the Bay Area.
6 of them will test positive.
One of them is guilty.

\[
p(\text{guilty} | \text{true test}) = \frac{1}{6}
\]

\[
p(\text{pregnant} | \text{woman}) = 2%
\]
\[
p(\text{woman} | \text{pregnant}) = 1
\]
I typed "statistics" into a search engine.
what did I mean to type?

Assume for now that I have a list of possible corrections.

A = I typed "statistics"
B = I meant to type __

\[ p(\text{meant to type } | \text{ typed "statistics"}) \]

\[ = \frac{p(\text{typed "statistics"} | \text{meant to type } \_\_)}{p(\text{typed "statistics"})} \times p(\text{meant to type } \_\_). \]

\[ p(\text{typed "statistics"} | \text{meant to type } \_\_) \sim \text{likelihood} \]

\[ p(\text{meant to type } \_\_). \]

\[ \text{derived by counting words in a large corpus.} \]
likelihood : error model.
edit distance : how many deletions, transpositions, alterations, insertions

\[
p(\text{word } A \mid \text{observing word } B) = 0.6 \quad \text{if edit distance } =
\]
0.25 : 1
0.10 : 2
0.05 : 3
0 : > 0

"statistics"

statistics - one insertion
statics - 3 deletions
statistic - 2 insertion, 1 deletion
statist. - 3 del. + 1 insertion
statistical - 1 insertion, 1 deletion
statistician...
\[ P(\text{each of these possible corrections}) \] \[ = P(\text{statistics}) \]

\[ = 0.25 \times \frac{2209}{100,000,000} \]

\[ = 0.05 \times \frac{8}{100,000,000} \]

\[ = 0.05 \times \frac{18}{100,000,000} \]

\[ = 0.1 \times \frac{2074}{100,000,000} \]

\[ = P(\text{statistics}) \]
Study of the Therapeutic Effects of Intercessory Prayer (STEP) in cardiac bypass patients: A multicenter randomized trial of uncertainty and certainty of receiving intercessory prayer

Herbert Benson, MD, Jeffery A. Dusek, PhD, Jan B. Sherwood, RN, Peter Lam, PhD, Charles F. Bethea, MD, William Carpenter, MDIV, Sidney Levitsky, MD, Peter C. Hill, MD, Donald W. Clem, Jr, Manoj K. Jain, MD, MPH, David Drumel, MDIV, Stephen L. Kopecky, MD, Paul S. Mueller, MD, Dean Marek, Sue Rollins, RN, MPH, and Patricia L. Hibberd, MD, PhD

Boston, MA; Oklahoma City, OK; Washington, DC; Memphis, TN; and Rochester, MN

Background Intercessory prayer is widely believed to influence recovery from illness, but claims of benefits are not supported by well-controlled clinical trials. Prior studies have not addressed whether prayer itself or knowledge/certainty that prayer is being provided may influence outcome. We evaluated whether (1) receiving intercessory prayer or (2) being certain of receiving intercessory prayer was associated with uncomplicated recovery after coronary artery bypass graft (CABG) surgery.

Methods Patients at 6 US hospitals were randomly assigned to 1 of 3 groups: 604 received intercessory prayer after being informed that they may or may not receive prayer; 597 did not receive intercessory prayer after being informed that they may or may not receive prayer; and 601 received intercessory prayer after being informed they would receive prayer. Intercessory prayer was provided for 14 days, starting the night before CABG. The primary outcome was presence of any complication within 30 days of CABG. Secondary outcomes were any major event and mortality.

Results In the 2 groups uncertain about receiving intercessory prayer, complications occurred in 52% (315/604) of patients who received intercessory prayer versus 51% (304/597) of those who did not (relative risk 1.02, 95% CI 0.92-1.15). Complications occurred in 59% (352/601) of patients certain of receiving intercessory prayer compared with the 52% (315/604) of those uncertain of receiving intercessory prayer (relative risk 1.14, 95% CI 1.02-1.28). Major events and 30-day mortality were similar across the 3 groups.

Conclusions Intercessory prayer itself had no effect on complication-free recovery from CABG, but certainty of receiving intercessory prayer was associated with a higher incidence of complications. (Am Heart J 2006;151:934-42)

More than 350,000 Americans and 800,000 people worldwide have coronary artery bypass graft (CABG) surgery every year. Despite advances in surgical techniques, anesthesia, and postoperative care in recent years, major and minor complications occur within 30 days of CABG (1997 Society of Thoracic Surgeons Adult Cardiac Surgery Database). Patients undergoing CABG often report that they are depressed, and depression is associated with cardiac events and mortality after CABG. Many patients report using private or family prayer to cope with this stressful experience.

Although the effects of private prayer on outcome after CABG are unknown, 4 trials investigated the effects of intercessory prayer in heterogeneous groups of cardiac patients. Results have been mixed — intercessory prayer was beneficial in 2 studies and had no effect in 2 studies.
Relative Risk.

\[ P_e - \text{incidence rate of some characteristic in the treatment group.} \]
\[ P_c - \text{in the control group.} \]

\[ RR = \frac{P_e}{P_c} \]

<table>
<thead>
<tr>
<th></th>
<th>complications</th>
<th>no complications</th>
</tr>
</thead>
<tbody>
<tr>
<td>treatment (pramy)</td>
<td>315</td>
<td>289</td>
</tr>
<tr>
<td>control (no pramy)</td>
<td>304</td>
<td>293</td>
</tr>
</tbody>
</table>

\[ RR = \frac{\text{# +ve in treatment group}}{\text{total # in treatment group}} = \frac{315}{604} = 0.524 \]
\[ RR = \frac{\text{# +ve in control group}}{\text{total # in control group}} = \frac{304}{597} = 1.024 \]
In this case the risk is the same for both groups.

\[ RR < 1 \quad \text{fraction by which the rate in the control group is reduced in the treatment group} \]

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\[ RR = \frac{1.12}{1} \]

Those who know they are being prayed for are 12\% more likely to have complications than those who don't know.

Be careful if both \( P_e \) and \( P_c \) are very small - the relative risk may be large, but the absolute risk may be so small it doesn't matter.
Odds: \[
\frac{\text{odds in favour of event } A}{p(A)}
\]

odds ratio: \[
\frac{\text{odds in favour of the event for treatment group}}{\text{odds in favour for the control group}}
\]

Rate: frequency per some reference size group (eg per 1000)

rate = relative freq. x reference size

Infant mortality is usually given as per 1000

\[
\frac{\text{# deaths of infants in 1st year}}{\text{# live births}} \times 1000
\]