**QUIZ QUESTION**

- On any sample, what is the largest proportion of observations that can fall 4 standard deviations away from the mean?
  - (a) $15/16$.
  - (b) $7/8$.
  - (c) $24/25$.
  - (d) $1/16$.
  - (e) $1/4$.

---

**Chebyshev's Inequality!**

$k = 4$

\[
\text{Prob} = 1 - \frac{1}{K^2} = 1 - \frac{1}{4^2} = \frac{15}{16}
\]

\[
1 - \frac{15}{16} = \frac{1}{16}
\]

\[
1 - \frac{1}{16} = \frac{16-1}{16} = \frac{15}{16}
\]
QUIZZ QUESTION

A population has a mean of 20 and a variance of 25. The z-score corresponding to an observation of 25 is

a) -3  
b) 1  
c) 1.5  
d) -1  
e) 0

\[ z = \frac{25 - 20}{5} = \frac{5}{5} = 1 \]

\[ \mu = 20 \]
\[ \sigma^2 = 25 \Rightarrow \sigma = 5 \]
\[ z = \frac{x - \mu}{\sigma} \]
QUIZZ QUESTION

- The boxplots correspond to the ages of actors and actresses who have won the Oscar. Which of the following is not necessarily true.
  
  a) The median age of male actors is greater than the median age of actresses.
  b) At least 75% of the actresses who have won the Oscar were under 45 years of age.
  c) At least 25% of the male actors who have won Oscars have been over 45.
  d) The mean age of male actors is greater than the mean age of actresses.
Cluster sampling: pick 2 out of 3 classrooms, and interview all students.

1) G1 - G2 \ A B C D E F G
2) G1 - G3 \ A B C H I
3) G2 - G3 \ D E F G H I

Prob (A) = \( \frac{2}{3} \)  \ Prob (B) = \( \frac{2}{3} \)

Random sampling vs

Simple random sample of size \( n \)
EXPERIMENTS

- For the purpose of this class, an experiment is any process whose outcome cannot be known for sure beforehand.
- The goal of probability theory is to provide us with the language and tools to deal with the uncertainty in the outcome of the experiment.
EVENTS

- The **sample space** is the set of all possible outcomes of an experiment. It is usually denoted by $S$.

- An **event** is a subset of the sample space that is of interest to us, i.e., $A$ is an event if $A \in S$.

- Example 1:
  - $A$: it will rain tomorrow.
  - $B$: it will be 60 degrees tomorrow.

- Example 2:
  - $A$: a given person smokes.
  - $B$: a given person will develop lung cancer in the next 10 years.
Operations on Sets

- Events are just sets, so you can use the traditional set operations:
  - Intersection: $A$ and $B$ occur simultaneously. $A \cap B$
  - Union: either $A$ or $B$ or both occur. $A \cup B$
  - Complement: $A$ does not occur. $A^c$ or $\overline{A}$

$A$: it rains today
$B$: it is more than 60°F today

$A \cap B$: it rains and it is more than 60°F today

$A \cup B$: it rains or it is more than 60°F today.

$A^c = \overline{A}$: it does not rain today
\( \Omega \rightarrow \text{The outcome space} \)

\( A \cup B \)

\( A \cap B \)

\( A \cup B \)

\[ A^c : \text{everything but A} \]

\( (A \cup B)^c \)

De Morgan's Law

\( A^c \cap B^c \) Not A and not B

\( (A \cap B)^c \) Not in A or B
PROBABILITY

- Frequentist definition of probability: If an experiment is repeated \( n \) times under essentially the same conditions, if \( A \) occurs \( m \) times then as \( n \) grows large \( m/n \) approaches to a fixed limit that is called the probability of \( A \).

\[
P(A) = \frac{m}{n}
\]

- This definition is easy to apply when thinking about experiments that are simple such as rolling a die, tossing coins etc but, how does it apply to processes such as predicting the weather or deciding whether a company has discriminated someone?
PROBABILITY

- It is hard to imagine how these processes can be repeated again and again independently and under the same conditions.
- It is more useful to think of chance or probability as a degree of belief of how likely something is to happen. $P(A)$ is estimated by using knowledge of the relevant circumstances.

Intrade
Basic Properties

- Probabilities are always between 0 and 1.
  \[ 0 \leq P(A) \leq 1 \]
- The probability that something will happen is 1.
  \[ P(S) = 1 \]
- The probability that something happens is 1 minus the probability that the opposite happens.
  \[ P(A) = 1 - P(\bar{A}) \]
OTHER RELEVANT CONCEPTS

- The **null event**, denoted by $\emptyset$, is the event that can never occur.

$$A \cap \overline{A} = \emptyset \Rightarrow P(A \cap \overline{A}) = P(\emptyset) = 0 \quad P(A \cup \overline{A}) = 1$$

- Two events $A$ and $B$ are said to be mutually exclusive if they cannot occur simultaneously. For example, $A$: a woman dies before she is 50, $B$: the same woman dies after she is 70. $A$ and $B$ are mutually exclusive. In this case,

$$P(A \cup B) = P(A) + P(B)$$

- A lot of these properties are easier to visualize using Venn diagrams.

$A$ and $B$ mutually exclusive

$$\Rightarrow A \cap B = \emptyset$$

$P(A \cup B) = P(A) + P(B)$
**Addition Rule**

- If $A$ and $B$ are not mutually exclusive then
  
  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- This can be generalized to more sets (this is called the *inclusion-exclusion principle*). For three of them:
  
  $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

\[ 
\begin{array}{c}
\text{You are counting it twice}
\end{array} 
\]
DE MORGAN'S LAW

- How are the three operations related?

\[
P(A \cup B) = 1 - P(A \cup B) = P(A \cap B) \\
P(A \cap B) = 1 - P(A \cap B) = P(A \cup B)
\]

\[
(P(A \cup B)^c) = P(A^c \cap B^c)
\]

not (A or B) := A does not happen and B does not happen
QUESTION

A review panel ran a retrospective study of newborn
discharge and rehabilitation, and the results are presented
in the table. If an infant is randomly selected, what is the
probability that it was rehospitalized within a week or it
was an early discharge.

- 6319/6776.
- 2860/6776.
- 3916/6776.
- 2860/6516.
- 457/6776.

<table>
<thead>
<tr>
<th></th>
<th>Rehospitalized within a week</th>
<th>Not rehospitalized within a week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early discharge</td>
<td>457</td>
<td>3199</td>
</tr>
<tr>
<td>Late discharge</td>
<td>260</td>
<td>2860</td>
</tr>
</tbody>
</table>

\[ A = \text{Early discharge} \implies A^c = \text{Late disch} \]
\[ B : \text{Rehospitalized within a week} \]

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]
\[
P(A \cap B) = \frac{457}{6776} \quad P(A) = \frac{3656}{6776}
\]
\[
P(B) = \frac{717}{6776}
\]

\[
P(A \cup B) = \frac{717 + 3656 - 457}{6776}
\]
QUESTION

In the same example, if an infant is randomly selected, what is the probability that it was rehospitalized within a week and it was an early discharge.

a) $6319/6776$.

b) $2860/6776$.

c) $3916/6776$.

d) $2860/6516$.

e) $457/6776$.

<table>
<thead>
<tr>
<th></th>
<th>Rehospitalized within a week</th>
<th>Not rehospitalized within a week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early discharge</td>
<td>457</td>
<td>8199</td>
</tr>
<tr>
<td>Late discharge</td>
<td>260</td>
<td>2860</td>
</tr>
</tbody>
</table>
**Question**

In the same example, if an infant is randomly selected, what is the probability that it was neither rehospitalized within a week nor was an early discharge.

- a) $\frac{6319}{6776}$.
- b) $\frac{2860}{6776}$.
- c) $\frac{3916}{6776}$.
- d) $\frac{2860}{6516}$.
- e) $\frac{457}{6776}$.

<table>
<thead>
<tr>
<th>Rehospitalized within a week</th>
<th>Not rehospitalized within a week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early discharge</td>
<td>457</td>
</tr>
<tr>
<td>Late discharge</td>
<td>260</td>
</tr>
</tbody>
</table>

\[
P(A^c \cap B^c) = \frac{2860}{6776}
\]

\[
P(A^c \cap B^c) = 1 - P(A \cup B) = \frac{2860}{6776}
\]