Quiz 3

Marine biologists have noticed that the color of the outermost growth band on a clam tends to be related to the time of the year in which the clam dies. A biologist conducted a small investigation of whether this is true for the species *Protothaca staminea*. She collected a sample of 66 clam shells from this species and classified them according to (1) month when the clam died and (2) color of the outermost growth band. The data are shown in the following table.

<table>
<thead>
<tr>
<th>Month/Color</th>
<th>Clear</th>
<th>Dark</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td>9</td>
<td>26</td>
</tr>
<tr>
<td>March</td>
<td>6</td>
<td>25</td>
</tr>
</tbody>
</table>

1. Compute the estimated proportion of clear clam shells out of the clams that died in February. (Denote this estimated proportion by $\hat{p}_{C,F}$).

**Answer.** $\hat{p}_{C,F} = \frac{9}{35} = 0.257$

2. Compute the estimated proportion of clear clam shells for the clams that died in March. (Denote this estimated proportion by $\hat{p}_{C,M}$).

**Answer.** $\hat{p}_{C,M} = \frac{6}{31} = 0.194$

3. Assume we are interested in testing the null hypothesis that the proportions of clear clam shells are identical in the months of February and March against the alternative hypothesis that they are not equal.

(a) Write down $H_0$ and $H_a$.

$H_0 : p_{C,F} = p_{C,M}$ against $H_a : p_{C,F} \neq p_{C,M}$
(b) Using $\alpha = 0.05$, conduct such test using the appropriate z-statistics, that is,

$$z = \frac{(\hat{p}_{C,F} - \hat{p}_{C,M})}{\sqrt{\hat{p}_C(1 - \hat{p}_C)(1/n_F + 1/n_M)}}$$

with $n_F$ and $n_C$ the total number of clam shells in the sample that died in February and March, respectively, and

$$\hat{p}_C = \frac{n_F\hat{p}_{C,F} + n_M\hat{p}_{C,M}}{n_F + n_M}$$

**Answer.** $n_F = 35$, $n_M = 31$ and therefore

$$\hat{p}_C = \frac{35 \times 0.257 + 31 \times 0.194}{35 + 31} = 0.227.$$ 

Now,

$$z = \frac{0.257 - 0.194}{\sqrt{0.227(1 - 0.227)(1/35 + 1/31)}} = \frac{0.063}{\sqrt{0.011}} = 0.601.$$ 

The critical value for the test is 1.96 and therefore, 0.601 does not lie in the rejection region so we do not have enough evidence to reject $H_0$ in favor of $H_a$. Equivalently, the p-value is $2 \times 0.271 = 0.542$, and since $\alpha < p$-value we do not have enough evidence in the data to reject $H_0$ in favor of $H_a$.

(c) What do you conclude? **Answer** Based on this sample, we cannot conclude that the proportions of clear clam shells are different in February and March.