Today: Probability
Mon: Probability Models for Sums

HW 1 & 2 have been graded too harshly.
1) The grades on HW 1 & 2 will be rescaled upward.
2) If you got less than \( \frac{1}{2} \) credit on any part of any problem you attempted (on HW 1 or 2), turn your paper in to your TA for regraded.

Solutions to HW 1 & quizzes 1 & 2 are posted in glass case near white boards in Baskin (near EE 126).

DD extra office hours today 3pm - 4pm

Equally Likely Model (ELM): if all the different ways experiment you're interested in could come out are equally likely, then

\[ P(A) = \frac{\text{(# of ways favorable to } A)}{\text{(# of ways experiment could come out)}} \]

1) \( 0\% \leq P(A) \leq 100\% \)

\( (a) \) and \( (i) \)

\[ P(A) + P(\text{not } A) = 100\% \]

Valuable calculation Device: \( P(A) = 100\% - P(\text{not } A) \) (easy rule)

Look at Case Study 7

\[ P(1 \text{ or more TS babies in family of } 5) = P(\text{exactly 1 TS baby or exactly 2 TS babies}) \]

\[ = 100\% - P(\text{no TS babies}) \]

\[ P(\text{no TS baby}) = P(\text{not TS on first and not TS on 2nd and \ldots and not TS on 5th}) \]

Basic Rules involve \( \neg, \lor, \land, \Box \text{ given} \)
\[ P(A \text{ or } B) = P(A) + P(B) \text{ but this is only a}
\text{special case when there is no overlap: } A \text{ and } B \text{ are mutually exclusive.} \]

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

**General addition rule for events**

\[ P(1 \text{ TS baby}) = P(\text{exactly } 1) + P(\text{exactly } 2) + \ldots + P(\text{exactly } 5) \]

No overlap:

\[ P(\text{exactly } 1) + P(\text{exactly } 2) + \ldots + P(\text{exactly } 5) = P(\text{No TS on } 1^\text{st} \text{ and } \text{No TS on } 2^\text{nd} \text{ and } \ldots \text{ and } \text{No TS on } 5^\text{th}) \]

I.I.D Sampling (Independent Identically Distributed)

\[ P(y_1 = 2 \text{ and } y_2 = 2) = \frac{1}{9} = \frac{1}{3} \]

\[ \frac{1}{3} \cdot \frac{1}{3} = P(y_1 = 2) \cdot P(y_2 = 2) \]

\[ \begin{array}{c|c|c|c}
 \text{Y_1} & \text{Y_2} & \text{Y_2} \\
 \hline
 1 & (1,1) & (1,1) \\
 2 & (1,2) & (2,2) \\
 3 & (2,1) & (2,1) \\
 4 & (3,1) & (3,1) \\
\end{array} \]

4 one (2,2) out of 9 possibilities

\[ P(y_1 = 2) = \frac{3}{9} = \frac{1}{3}; \quad P(y_2 = 2) = \frac{3}{9} = \frac{1}{3} \]

Sometimes: \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

\[ P(y_1 = 2 \text{ and } y_2 = 2) = \frac{1}{9} \]

\[ P(y_1 = 2) = \frac{2}{6} = \frac{1}{3} \]

\[ P(y_2 = 2) = \frac{2}{6} = \frac{1}{3} \quad \text{So here } P(y_1 = 2 \text{ and } y_2 = 2) \neq P(y_1 = 2) \cdot P(y_2 = 2) \]

\[ \text{Product rule does not apply w/ SRS because the 2 draws w/ SRS are independent.} \]
P(B given A) = \frac{P(A \text{ and } B)}{P(A)}

Therefore \( P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A) \)

P(B) = \frac{13}{4 - 1}

\text{General Product Rule for (And)}

\begin{align*}
\text{In special case that } A \text{ and } B \text{ are independent ('knowledge of one doesn't affect the other')} & \\
P(B \text{ given } A) = P(B), \ P(A \text{ given } B) = P(A) & \text{ general product rule simplifies to } P(A \text{ and } B) = P(A) \cdot P(B)
\end{align*}

P(y_1 = 2 \text{ and } y_2 = 2) = P(y_1 = 2) \cdot P(y_2 = 2 \text{ given } y_1 = 2) = \frac{1}{3} \cdot 0.75 = 0.25

P(\text{No TS Babies}) = P(\text{Not TS on 1st and Not TS on 2nd and ... and Not TS on 5th})

\begin{align*}
\text{Independent } & \\
& = P(\text{Not TS on 1st}) \cdot P(\text{Not TS on 2nd}) \cdot \ldots \cdot P(\text{Not TS on 5th}) \\
& = (1 - 0.14) \cdot (1 - 0.14) \cdot \ldots \cdot (1 - 0.14) \\
& = (1 - 0.14)^5 = \left( \frac{9}{10} \right)^5 = 0.24
\end{align*}

So finally, \( P(\text{1 or more}) = 100\% \cdot P(\text{No TS}) \)

\( = 100\% \cdot 24\% = 76\% \)
Suppose I choose 1 person at random from these 106 people. ELM.

\[ P(Y) = \frac{81}{106} = 76\% \]
\[ P(M) = \frac{52}{106} = 50\% \]
\[ P(Y \text{ and } M) = \frac{52}{106} = 49\% \]

Q: Are gender & MLP independent?

\[ P(Y) = 76\% \]
\[ P(Y \mid M) = \frac{82}{52} = 91\% \quad \text{No, So Gender & MLP are} \]
\[ P(Y \mid F) = \frac{29}{49} = 59\% \quad \text{strongly dependent} \]