Class Notes

- This time: probability
- Next time: probability models for sums

- Reading: DD ch 10
  - FPP ch 17 (25%)

- HW #1 & #2 have been graded too harshly
  - They will be rescaled upward
  - If you received less than ½ credit on any part of any problem attempted, turn into your TA for regading

- HW #1 and quizzes #1 & #2 solutions are posted in glass case near white boards in Baskin near BE 126
- Take all grading questions to Ethan (head TA) at BE 142
- DD extra office hour today, at Baskin white boards (3-4 pm)

Equally Likely Model (ELM)

If all possible outcomes are equally likely then:

\[ P(A) = \frac{\# \text{ of favorable to } A}{\# \text{ of possible outcomes}} \]

Checkerboard:

<table>
<thead>
<tr>
<th>A</th>
<th>not A</th>
</tr>
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Easy Rules

1. \(0\% \leq P(A) \leq 100\%\)
2. \(P(A) + P(\text{not } A) = 100\%\)

Valuable calculation device:

\[ P(A) = 100\% - P(\text{not } A) \]
The probability of having one or more Tay-Sachs babies in a family of 5 is given by:

\[ P(1 \text{ or more TS babies in family of 5}) = P(1 \text{ or 2 or...or 5}) \]

\[ = 100\% - P(\text{no TS babies}) = 100\% - P(\text{not 1st and not...and not } 5^{th}) \]

- Basic rules: \( \neg \) (defined above), \( \lor \), \( \land \), \( \circ \), \( \circledast \) given:

**OR**

For this case only (mutually exclusive):

\[ P(A \text{ or } B) = P(A) + P(B) \]

- For the general case:

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

\[ P(1 \text{ or more TS}) = P(\text{exactly 1 or...exactly 5}) = P(\text{exactly 1}) \]

\[ + P(\text{exactly 2}) + P(\text{exactly 3}) + ... + P(\text{exactly 5}) = 100\% \]

\[ = P(\text{no TS babies}) = P(\text{not 1st and...and not } 5^{th}) \]

**AND**

\[ p^2 \]

\[
\begin{array}{c|ccc}
1 & 1,1 & 1,2 & 1,9 \\
2 & 2,1 & 2,2 & 2,9 \\
9 & 9,1 & 9,2 & 9,9 \\
\end{array}
\]

\[ ELM \text{ applies, so} \]

\[ p(y_1 = 2 \text{ and } y_2 = 2) = \frac{1}{4} \]

\[ = p(y_1 = 2) \times p(y_2 = 2) \]

\[ = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \]
for SRS (at random w/ out replacement)

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| 1 | 2 | 9 | y₁
| 2 | 2 | 1 | y₂
| 9 | 1 | 2 |

\[ P(y₁ = 2 \text{ and } y₂ = 2) = 0 \]

\[ P(y₁ = 2) = \frac{2}{9} = \frac{1}{3} \]
\[ P(y₂ = 2) = \frac{2}{9} = \frac{1}{3} \]

\[ P(y₁ = 2 \text{ and } y₂ = 2) \neq P(y₁ = 2) \times P(y₂ = 2) \]

So earlier product rule does not apply with SRS because 2 draws with SRS are dependent.

\[ P(B \text{ given } A) = ? \]
Conditional probability of B given A
\[ P(B \mid A) \]

\[ P(B) = \frac{B}{A+B} \leq 0 \]

\[ P(B \text{ given } A) = \frac{0}{A} = \frac{P(A \text{ and } B)}{A} \]

(Thomas Bayes, 1720):

Therefore: General Product Rule for \(AND\)

\[ P(A \text{ and } B) = P(A) \times P(B \text{ given } A) \]
\[ = P(B) \times P(A \text{ given } B) \]

In the special case where A and B are independent, \( P(B \mid A) = P(B) \) \& \( P(A \mid B) = P(A) \), so general product rule simplifies to

\[ P(A \text{ and } B) = P(A) \times P(B) \]

Ex: \( P(y₁ = 2 \text{ and } y₂ = 2) = P(y₁ = 2) \times P(y₂ = 2 \text{ given } y₁ = 2) \)
\[ = \frac{1}{3} \times 0 = 0 \quad \checkmark \text{ worked!} \]
[Tay Sachs CS]
\[ P(\text{no TS babies}) = P(\text{not 1st}) \text{ and } \ldots \text{ and } P(\text{not 5th}) \]
- independent so multiply
\[ = p(\text{not 1st}) \times \ldots \times p(\text{not 5th}) = (1 - \frac{1}{2})^5 = 0.75^5 = 0.24 \]
\[ \Rightarrow P(\text{at least 1 TS baby}) = 1 - 0.24 = 76\% \]

[practice with conditional probability]
MLP - marijuana legalization preference

MLP  | gender
-----|------
\[ \begin{array}{c|c}
\text{M} & \text{F} \\
\hline
\text{y} & \text{y} \\
\text{M} & \text{F} \\
\hline
\text{F} & \text{M} \\
\end{array} \]
\[ n = 106 \rightarrow \text{sort} \]

\[ \begin{array}{c|c|c}
\text{M} & \text{MLP} & \text{table} \\
\hline
\text{y} & \text{F} & 29 \\
\text{y} & \text{F} & 52 \\
\text{n} & \text{F} & 20 \\
\text{n} & \text{F} & 5 \\
\end{array} \]

Suppose I choose 1 person at random from these 106 people - ELM? \(\square\)

\[ P(y) = \frac{29}{106} = 76\% \]
\[ P(M) = \frac{52}{106} = 54\% \]
\[ P(y \text{ and } M) = \frac{29}{106} = 19\% \]

Q: Are MLP and gender independent here?
\[ P(y) = 76\% \]
\[ P(y | M) = 91\% \]
\[ P(y | F) = 59\% \]

A: no, they are quite dependent.