Class Notes

this time: Comparing 2 Samples
next time: Comparing 2 Samples
- reading: FPP ch. 97
- homework 5 (p. 107-109 in reader) due Fri, Jun 2, 2006

[case study 13 1/2] - t-test

\[
\frac{\bar{y} - \mu}{s/\sqrt{n}} \text{ if null true}
\]

\[
e = \frac{77.9 - 70}{8.1/\sqrt{5}} = 2.16
\]

\[
\begin{array}{c|c|c}
78 & \uparrow & \text{Evidence running against null hypothesis, but not very strongly. Borderline}\n83 & \downarrow & \text{necessity to recalibrate.}\n68 & \downarrow & \text{necessary to recalibrate.}\n72 & \downarrow & \text{necessary to recalibrate.}\n88 & \downarrow & \text{necessary to recalibrate.}\n\end{array}
\]

\[
\text{mean (y)} = 77.8
\]

\[
\text{SD (s)} = 8.1
\]

Q: Why are the (n-1) degrees of freedom with a sample of size n?

A: whole premise: use \( \bar{y} \) to estimate \( \mu \), this is like holding
\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_n
\end{bmatrix}
\]

\[
\text{mean} = 77.8
\]

n = 5, 5 numbers are involved, but only
\( y (5-1) \) of them are free to vary.
A pitfall of significance testing:

\[
\begin{align*}
(\text{statistical \hspace{1cm}}) & \neq (\text{practical \hspace{1cm}}) \\
(\text{significance \hspace{1cm}}) & \neq (\text{significance})
\end{align*}
\]

Ex: New drug to combat high blood pressure:

\begin{itemize}
  \item \text{Null hyp (} H_0 \text{): drug doesn't work}
  \item \text{Alt hyp (} H_1 \text{): } \mu \neq 0
\end{itemize}

\[
\begin{align*}
\text{mean (} \bar{y} \text{)} & = -1 \text{ mm Hg} \\
\text{SD (} s \text{)} & = 20 \text{ mm Hg}
\end{align*}
\]

\[
\begin{align*}
\hat{\mu} & = \bar{y} = -1 \\
SE (\bar{y}) & = \frac{s}{\sqrt{n}} = \frac{20}{\sqrt{50}} = 2.83
\end{align*}
\]

\[
\begin{align*}
Z & = \frac{\bar{y} - \mu_0}{SE (\bar{y})} = \frac{-1 - 0}{2.83} = -4.5
\end{align*}
\]

\[
\begin{align*}
\text{CLT} & \quad \text{(highly statistically significant)}
\end{align*}
\]

\[
\begin{align*}
p & = 0.00
\end{align*}
\]

Q: How did this happen?

- \text{Statistically because } \rho \text{ is small}
- \text{p is small because } \rho \text{ is big}
- \text{Z is big in (} -1.64 \text{) when numerator is big or denominator is small.}
- \text{SE (denominator) is small when (} \frac{s}{\sqrt{n}} \text{)}
- \text{fraction is small (num. is small or den. is huge)}
- \text{Here there is too large of a } n
  \text{ (too much data.)}
an example with too little data:

<table>
<thead>
<tr>
<th>After</th>
<th>Before</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{mean}(\text{after}) = \text{10 mm Hg} \\
\text{SD} (5) = \text{20 mm Hg}
\]

\[
\frac{z}{(t_{(0.05, 0.4)})} = -1.4 \\
\frac{\text{S.E.}(t_{(0.05, 0.4)})}{\text{20}} = 7 \\
\text{L.R.} \text{test } t^2 \text{ H}_{0} \text{ is true.}
\]

\[
\text{S.E.} = 7 \\
\text{L.R.} (1) \text{ is between -24 and 14.} \\
\text{Thus it, however, rejects } (-10 \text{ mm Hg is good}).
\]

\[
\text{at } \text{19} \text{ and } \text{34.} \\
\text{85% CI: } -16 \pm 14 = \frac{(\text{mean} - \text{SD})}{\sqrt{\text{24}}} \\
\text{Limiting as } 0 \text{ is between } -24 \text{ and } 34.
\]

\[
\text{FIX FOR THIS PITFALL: make sure } n \text{ is right so } \\
\text{STATISTIC } \equiv \text{PRACTICAL}
\]

VI. VII: comparing 2 samples

2 Topics:

1) paired comparisons

2) analysis of 2 independent samples

1 - PAIRED comparisons: [Case Study /4]

outcome: sales volume (in cases) of product treatment: discount \( T \) vs standard \( C \)

marketing plan

one possible design: get a bunch of stores (ex. \( n = 120 \))

Randomize 60 to \( T \) and 60 to \( C \).

\[
\text{mean}(T) \quad \text{mean}(C)
\]

\[
\text{estimate of treatment effect} = (g_1 - g_2)
\]
Q: Better design possible?
   PCFs? Randomization will make treatment and control groups similar on PCFs.

A: But can something be done to increase the efficiency (accuracy)?
A: Yes, think about PCFs. High PCF is the overall sales of all products. Stores that sell more of everything will show to sell more of any one product.

Q: How can we improve design by using this PCF?

A: PCF

<table>
<thead>
<tr>
<th>High</th>
<th>857</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>787</td>
<td>699</td>
</tr>
<tr>
<td>Med</td>
<td>814</td>
<td>812</td>
</tr>
</tbody>
</table>

MATCH PAIRS DESIGN:
- Hold PCF constant in creating match pairs of stores. Assign to + and 0 at random.

Design 2 is also valid but will be more efficient because PCF was controlled in design.