AMS 5 (Statistics): Extra discussion section problem

(public policy) Since the early 13th century, coins struck by the Royal Mint in England have been evaluated for their metal content on a sample basis, in a ceremony called the *Trial of the Pyx*. This ceremony does not have much meaning anymore, but there was real money on the line back in the 1700s, because English coins were made of gold in those days. In 1799, for instance, the procedure went like this. 100 gold coins called guineas were chosen at random from all of the coins made at the Mint that year, put in the Pyx (a ceremonial box), and weighed. The Master of the Mint, who was responsible for the quality of the coins, was allowed a margin of error, called the “remedy,” which was set according to the manufacturing tolerances of the time.

In 1799 a guinea was supposed to weigh 128 grains (there are 360 grains in an ounce), so the 100 guineas in the Pyx should have weighed about 12,800 grains. The remedy in those days was $\frac{1}{400}$ of the expected amount, or 32 grains. If the actual weight of the coins in the Pyx differed from its expected value by more than the remedy on either the high or low side, the Master of the Mint was exposed to serious penalties. The British government had a vested interest in the coins not weighing too much, but the Master of the Mint had an incentive to make them weigh less than the standard, because he got to keep the shortfall himself (as long as he was not caught by the Trial of the Pyx).

(a) Build a probability model for this setup, being explicit about the population, sample, and imaginary datasets. If the Master of the Mint is honest and manufactures guineas that weigh exactly 128 grains on average, with an SD of 1 grain, what is the chance that he will survive the Trial of the Pyx? Show all your work.

(b) If instead he sets things up so that the guineas weigh only 127.7 grains on average (with the same SD of 1 grain), what is the chance now that he will survive the Trial? If he does survive, how much gold (in ounces) can he expect to pocket in an average year in which he produces 100,000 guineas? Give or take how much? Show all your work (but you don’t need to explicitly rebuild the model).