1 Manipulating negative signs

1.1 Removing brackets in an expression

Rule:

- Brackets preceded by a + sign can just be removed
- Brackets preceded by a − sign required some care: think of them as −1 × (expression). To remove the bracket, you need to distribute −1.

Examples:

- \((x^2 + 2x - 1) + (x + 3) = x^2 + 2x - 1 + x + 3 = x^2 + 3x + 2\)
- \((x^2 + 2x - 1) − (x + 3) = x^2 + 2x - 1 - x - 3 = x^2 + x - 4\)
- \(−(x^2 + 2x - 1) − (x + 3) = −x^2 - 2x + 1 - x - 3 = −x^2 - 3x - 2\)

Important: Bear in mind which bracket corresponds to which sign. For example,

\[−[2x + 2 − (x + 1)] = −2x − 2 + (x + 1)\]

and not \(−2x − 2 + (x − 1)\) (i.e. don’t change all the signs automatically).

Practice: Remove the brackets and simplify.

- \((x^3 + 3x^2 - 2) − (x - x^3) − (3x + 2x^2) = 2x^3 + x^2 - 4x - 2\)
- \(−(x + 1) + (4x^3 - 2x + 1) − [−(x + 2) + (x^2 + 1) − (x^3 + 2x - 1)] = 5x^3 - x^2\)
- \(3x + 2 − [(x + 2) + (x^3 - 3x + 1) − [−(x - 2) + (2 - 3x^2)]] = −x^3 - 3x^2 + 4x + 3\)
- \([3x + 2 - (x + 1)] - [2x + 1 - (3x^2 + 3x + 3)] = 3x^2 + 3x + 3\)

1.2 Adding brackets in an expression

Rule:

- You can always add brackets around any expression preceded by a + sign without changing anything.
- If you add brackets around an expression preceded by a − sign you have to change the sign of each term in the new bracketed expression.

Examples:

- \(x^2 - 2x + 1 = (x^2 - 2x + 1) = (x^2 - 2x) + 1 = x^2 - (2x - 1)\)
- \(−x^3 + 2x^2 - 3x - 1 = -(x^3 - 2x^2 + 3x + 1) = -(x^3 - 2x^2 + 3x) - 1 = -(x^3 - 2x^2) - (3x + 1) = -x^3 + (2x^2 - 3x - 1) = ...\)

Practice: Write these expressions at least 5 different ways using brackets in different places. Treat the various expression \((x - a)(x - b)\) in the second expression as "1 term" (i.e. do not expand them).
\begin{itemize}
\item \(-2x^3 - 2x^2 + x - 4\) = \(- (2x^3 + 2x^2) + (x - 4)\) = \(- (2x^3 + 2x^2 - x) - 4\)
\item \(-x(x - 1) + (x - 2)(x + 1) - 2(1 - x) + 1\) = \(- \left[ x(x-1) + (x-2)(x+1) \right] - \left[ 2(1-x) - 1 \right] \)
\end{itemize}

**Practice:** Combine these expressions into a single bracket, written as \([-\ldots]\), and then simplify inside this bracket (i.e., do not simplify first).
\begin{itemize}
\item \(- (x + 2) + (x - 1) = \ldots\) = \(- \left[ (x+2) - (x-1) \right] \) = \(- \left[ x + 2 - x + 1 \right] \) = \(-3\)
\item \((2x^2 + x - 1) - (2x^3 + x + 1) = \ldots\) = \(- \left[ (2x^2 + x - 1) + (2x^3 + x + 1) \right] \) = \(- \left[ 2x^3 - 2x^2 + 2 \right] \)
\item \(- (x + 2) - (x - 1)(x + 1) = \ldots\) = \(- \left[ (x+2)(x-2) + (x+1)(x-1) \right] \) = \(- \left[ 2x^2 - 5 \right] \)
\end{itemize}

## 2 Expanding polynomials

**Vocabulary:** Expanding a polynomial means to take whichever form it is currently in, and put it in the following form:

\[a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_0\]

**Example:** \((x + 1)(x - 2) + 3 = x^2 - 2x + x^2 - 2 + 3 = x^2 - x + 1\). There, the largest value of \(n\) is 2, and \(a_2 = 1\), \(a_1 = -1\) and \(a_0 = 1\).

**Important:** When dealing with polynomials with more than 2 factors multiplying each other, first multiply two factors and simplify that, then multiply the third one to the result. Note that it doesn’t matter which two terms you first choose.

**Example:**
\[
(\ x - 1)(2 - x^2)(3x + 1) = (2x - x^3 - 2 + x^2)(3x + 1)
= (-x^3 + x^2 + 2x - 2)(3x + 1)
= -3x^4 + 3x^3 + 6x^2 - 6x - x^3 + 3x^2 + 2x - 2
= -3x^4 + 2x^3 + 7x^2 - 4x - 2
\]

**Practice:** Expand the same polynomial two different ways by multiplying two other terms first.

**Important:** When dealing with polynomials containing brackets and factored polynomials inside these brackets, it’s usually worth simplifying first inside the bracket.

**Important:** Watch out for the signs!

**Example:**
\[
(x - 1)[3 - (x - 1)(x + 2)] = (x - 1)[3 - (x^2 + 2x - x - 2)]
= (x - 1)[3 - (x^2 + x - 2)]
= (x - 1)[3 - x^2 - x + 2]
= (x - 1)(-x^2 - x + 5)
= -x^3 - x^2 + 5x + x^2 + x - 5
= -x^3 + 6x - 5
\]

**Practice:** Expand the following polynomials. Be careful of the signs.
\begin{itemize}
\item \((x + 1)(x - 2) - (3x^2 + 1)(1 - x)\) = \(3x^3 - 2x^2 - 3\)
\item \(-2x(x - 1) + (2 - x)(x + 1)(3 - x)\) = \(x^3 - 6x^2 + 3x + 6\)
\item \(-3(2x^3 - 1) - 6x^2(x + 1) + 2x(x + 1)\) = \(-6x^4 + 18x^3 + 20x^2 + 8x\)
\item \((4x + 2)(2x - 1)(x+3) - 2) + (x + 1)[3x(x - 1) - 2x^2]\) = \(9x^3 + 22x^2 - 13x - 10\)
\end{itemize}