MATH 3 formula sheet

All formula with an asterisk must be known by heart. All others are optional and will be given to you in exams if required.

1 Lines, circles and points

The equation of a line with slope $s$ and y-intercept $b$ is

$$ y = f(x) = sx + b \quad (\ast) $$

(1)

If the line goes through 2 points A and B with coordinates $(x_A, y_A)$ and $(x_B, y_B)$ then

$$ s = \frac{y_B - y_A}{x_B - x_A} \quad (\ast) $$

(2)

The distance between two points $A(x_A, y_A)$ and $B(x_B, y_B)$ is

$$ d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \quad (\ast) $$

(3)

The equation of a circle of radius $R$ centered on $A(x_A, y_A)$ is

$$(x - x_A)^2 + (y - y_A)^2 = R^2 \quad (\ast) $$

(4)

2 Quadratic equations

$$ y = f(x) = ax^2 + bx + c \quad (\ast) $$

(5)

The graph of $y = f(x)$ is a parabola. It has a minimum (i.e. parabola opens upwards) if $a > 0$. It has a maximum (i.e. parabola opens downwards) if $a < 0$. The minimum/maximum is at the location $x_m$ with

$$ x_m = -\frac{b}{2a} \quad (\ast) $$

(6)

It has roots (i.e it intersects the x-axis) when $y = f(x) = 0$. The solutions to this equation depends on the value of $D$:

$$ D = b^2 - 4ac \quad (\ast) $$

(7)

- if $D < 0$ there are no solutions. The parabola does not intercept the x-axis.
  The function $f(x) = ax^2 + bx + c$ cannot be factored.
- if $D = 0$ there is one solution. The parabola just touches the x-axis at the point

$$ x_1 = x_m = -\frac{b}{2a} \quad (\ast) $$

(8)

The function $f(x) = ax^2 + bx + c$ is factored as

$$ f(x) = a(x - x_1)^2 \quad (\ast) $$

(9)
• if $D > 0$ there are two solutions. The parabola intercepts the $x$-axis in the two points

$$x_1 = \frac{-b - \sqrt{D}}{2a}, \quad x_2 = \frac{-b + \sqrt{D}}{2a} \quad (*) \quad (10)$$

The function $f(x) = ax^2 + bx + c$ is factored as

$$f(x) = a(x - x_1)(x - x_2) \quad (*) \quad (11)$$

3 Polynomial functions

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \quad (*) \quad (12)$$

$a_n$ is the leading coefficient, $n$ is the order of the polynomial.

The factored form of $f$ is

$$f(x) = a_n(x - x_1)(x - x_2)(x - x_3)\ldots(x - x_m)q(x) \quad (*) \quad (13)$$

where $x_i$ are all possible solutions to $f(x) = 0$ and $q(x)$ is a polynomial of order $n - m$, leading coefficient 1, with no roots ($q(x) \neq 0$).

4 Rational functions

$$y = f(x) = \frac{p(x)}{q(x)} \quad (*) \quad (14)$$

where $p(x)$ and $q(x)$ are polynomial functions.

The roots of $p(x)$ are the roots of $f(x)$. The roots of $q(x)$ are the asymptotes of $f(x)$.

5 Power functions

$$y = f(x) = x^a \quad (*) \quad (15)$$

Properties:

$$x^{a+b} = x^a x^b \quad (*) \quad (16)$$

$$x^{-a} = \frac{1}{x^a} \quad (*) \quad (17)$$

$$x^{a-b} = \frac{x^a}{x^b} \quad (*) \quad (18)$$

$$x^{ab} = (x^a)^b = (x^b)^a \quad (*) \quad (19)$$

6 Exponential functions

Exponential in base $a$:

$$y = f(x) = a^x \text{ with } a > 0 \quad (*) \quad (20)$$

Natural exponential (exponential in base $e$ with $e = 2.71828\ldots$):

$$y = f(x) = e^x = \exp(x) \quad (*) \quad (21)$$
Properties of all exponential functions:

\[
\begin{align*}
    a^{x+z} &= a^x a^z \quad (*) \\
    a^{-x} &= \frac{1}{a^x} \quad (*) \\
    a^{x-z} &= \frac{a^x}{a^z} \quad (*) \\
    a^{xz} &= (a^x)^z = (a^z)^x \quad (*) 
\end{align*}
\]

(22) \ (23) \ (24) \ (25)

7 Logarithmic functions

Logarithm in base \(a\) is the inverse of the exponential in base \(a\):

\[
y = \log_a(x) \text{ is equivalent to } x = a^y \quad (*)
\]

(26)

Natural logarithm (logarithm in base \(e\)) is the inverse of the natural logarithm:

\[
y = \ln(x) \text{ is equivalent to } x = e^y \quad (*)
\]

(27)

Inverse relations:

\[
\begin{align*}
    \log_a(a^x) &= x \quad (*) \\
    a^{\log_a(x)} &= x \quad (*) \\
    \ln(e^x) &= x \quad (*) \\
    e^{\ln(x)} &= x \quad (*) 
\end{align*}
\]

(28) \ (29) \ (30) \ (31)

Properties of all logarithmic functions (where \(a\) is a positive constant).

\[
\begin{align*}
    \log_a(xy) &= \log_a(x) + \log_a(y) \quad (*) \\
    \log_a\left(\frac{1}{x}\right) &= -\log_a(x) \quad (*) \\
    \log_a\left(\frac{x}{y}\right) &= \log_a(x) - \log_a(y) \quad (*) \\
    \log_a(x^c) &= c \log_a(x) \quad (*) 
\end{align*}
\]

(32) \ (33) \ (34) \ (35)

Relations for changing bases:

- From an exponential function in base \(a\) to the natural exponential:

\[
a^x = e^{x \ln a} \quad (*)
\]

(36)

- From a logarithmic function in base \(a\) to the natural logarithm:

\[
\log_a(x) = \frac{\ln x}{\ln a} \quad (*)
\]

(37)
8 Trigonometric functions

The basic trigonometric functions are:

\[ y = f(x) = \sin(x) \quad (\ast) \]
\[ y = f(x) = \cos(x) \quad (\ast) \]
\[ y = f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} \quad (\ast) \]
\[ y = f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)} \quad (\ast) \]
\[ y = f(x) = \sec(x) = \frac{1}{\cos(x)} \quad (\ast) \]
\[ y = f(x) = \csc(x) = \frac{1}{\sin(x)} \quad (\ast) \]

Table of values you have you know (\ast):

<table>
<thead>
<tr>
<th>Angle (degree)</th>
<th>Angle (radian)</th>
<th>sin</th>
<th>cos</th>
<th>tan</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>(\pi/6)</td>
<td>0.5</td>
<td>(\sqrt{3}/2)</td>
<td>(1/\sqrt{3})</td>
</tr>
<tr>
<td>45</td>
<td>(\pi/4)</td>
<td>(\sqrt{2}/2)</td>
<td>(\sqrt{2}/2)</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>(\pi/3)</td>
<td>(\sqrt{3}/2)</td>
<td>0.5</td>
<td>(\sqrt{3})</td>
</tr>
<tr>
<td>90</td>
<td>(\pi/2)</td>
<td>1</td>
<td>0</td>
<td>not defined</td>
</tr>
</tbody>
</table>

Properties:

\[ \cos^2 x + \sin^2 x = 1 \quad (\ast) \]
\[ \sin(2x) = 2 \sin x \cos x \quad (\ast) \]
\[ \cos(2x) = \cos^2 x - \sin^2 x \quad (\ast) \]
\[ \tan(2x) = \frac{2 \tan x}{1 - \tan^2 x} \]

Other addition/multiplication formula

\[ \cos(a + b) = \cos a \cos b - \sin a \sin b \]
\[ \cos(a - b) = \cos a \cos b + \sin a \sin b \]
\[ \sin(a + b) = \sin a \cos b + \cos a \sin b \]
\[ \sin(a - b) = \sin a \cos b - \cos a \sin b \]
\[ \tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \]
\[ \tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b} \]