6.6 The inverse trigonometric functions

Textbook Section 6.1

6.6.1 Definitions

We define the three basic inverse trigonometric functions:

However, note how neither of the three basic trigonometric functions pass the horizontal line test. As a consequence, the domain of definition of the inverse functions is limited to a region where the function does pass the test.
Based on that, we can now graph the \( \sin(x) \) function and its inverse:

As well as the \( \cos(x) \) function and its inverse:

And finally, the \( \tan(x) \) function and its inverse:
As for any function and their inverse, \( f(f^{-1}(x)) = f^{-1}(f(x)) = x \) so, for any \( x \) within the domain of definition of the respective functions, we have:

**EXAMPLES:**

- What is \( \arcsin(\sin(\pi/3)) \)?
- What is \( \sin((\arcsin(2)) \)?
- What is \( \arctan(\tan(\pi/3)) \)?
- What is \( \tan((\arctan(2)) \)?

Finally, note that in Calculus, the expressions \( \sin(\cos^{-1}(x)) \) and \( \cos(\sin^{-1}(x)) \) will often come up. These cannot be simplified as easily as the other ones, but on the other hand, by combining them with the Pythagorean formula, we can still get a nicer formula:
6.6.2 Applications of inverse trigonometric functions

Example 1: A contractor is installing a door in a narrow corridor. The door is 3 ft long, centered in the wall, and the corridor is 4 feet wide (see diagram). What is the maximum angle this door could open?

Example 2: The water level of tides in Santa Cruz in Steamer’s lane is given by \( h(t) = 15 + 6 \cos \left( 2 + \frac{\pi}{6} t \right) \). At what time is the low tide?