Chapter 1

The notion of functions

Textbook Chapter 1

1.1 The concept of functions

Although the concept of a function was invented a very long time ago, it is very easy today to gain an intuitive notion of what functions are because of their natural role in most computer and/or web-based applications, in engineering, and in economics, etc.

Example 1A: Ordering diapers on Amazon

1 Diaper bag: $45

When ordering n bags: Price = 45n + tax + shipping

Tax rate 8.5% → 45n · 0.085 = tax

Price: 45n + 0.085 · 45n + 10 = 48.825n + 10 = p(n)

Example 1B: Buying tomatoes at Safeway, self-checkout

Tomatoes: $4/pound

Self checkout: weight → w (pounds)

Price: 4w + tax = 4w + 0.085 · 4w

= 4w (1 + 0.085) = 4.34w = p(w)

Example 2: Bank Accounts. CD accounts offer various interest rates depending on the amount you put in: for instance, as of 09/19/15, at Chase, for a 2-year fixed term deposit you get interests of:

• 0.15% per year for accounts under $10K

• 0.25% per year for accounts between $10K and under $100K.
\* 0.30\% per year for accounts between $100K and under $250K.

The banker uses a computer function to tell you what your gain after 2 years will be as a function of your initial investment (cf. https://www.bankofinternet.com/calculators/apy-interest-calculator):

\[
\text{If } x < 10,000 \quad \text{Interest rate: } 0.15\%.
\]

Money in account after 1 year: \( x + 0.0015x = 1.0015x \)
2 years: \( 1.0015x + 0.0015(1.0015x) \)
\[= 1.0015x (1+0.0015) \]
\[= 1.0015^2 x \]

Gain = \((1.0015)^2 x - x = [(1.0015)^2 - 1] x = 0.003 x\)

\[
\text{If } 10K < x < 100K : \quad \text{Interest rate 0.25\%}.
\]

\[
\text{Gain} = [(1.0025)^2 - 1] x = 0.005 x
\]

\[
\text{If } 100K < x \quad \text{Interest 0.30\%} \Rightarrow \text{Gain} = [(1.003)^2 - 1] x
\]
\[= 0.006 x\]

Example 3: Installing Cable TV (see Problem 22 page 105 of Textbook).

![Diagram of a house and road]

- Cost of installing along road: \(500 \text{ \$ per mile}\)
- Cost of installing off road: \(700 \text{ \$ per mile}\)

Question: What is cost as a function of \(x\)?

\[
\text{Cost} = \text{Cost off road} + \text{Cost along road}
\]
\[
= 700 \cdot D(x) + 500 \cdot x
\]

\[
D(x) = \sqrt{4 + (5-x)^2}
\]

\[
\text{Cost}(x) = 700\sqrt{4 + (5-x)^2} + 500 \cdot x
\]

1.2 Mathematical definitions

1.2.1 Definition of a function

**Definition:** A function.

This definition is much more general than what we have seen so far, and covers a much wider class of functions, as for instance in the following examples: