Problem 1: Short questions. [60 points]

Given the function $f(x) = \frac{2x^2 - 2}{x+1}$

1. What is its domain of definition? ANSWER:

2. Simplify $f(x)$, for $x$ inside the domain of definition. ANSWER: $f(x)$ = 

3. Sketch the function, making sure to annotate your graph.

4. What is the domain of $\ln(f(x))$? ANSWER:
5. Solve the inequality $\frac{7}{x+1} < 3$. ANSWER: $x \in$ ________________

Given the functions $f(x) = e^x$ and $g(x) = 3 \ln(x)$

6. What is $f \circ g(x)$? _______________________________ (Simplify if possible)

7. What is $g \circ f(x)$? _______________________________ (Simplify if possible)

9. Given the function $f(x) = 3x + 4$, what is $f^{-1}(x)$? ANSWER: $f^{-1}(x) =$___________________

10. Given the function $f(x) = \sqrt{x^3 - 1}$, what is $f[f^{-1}(x^2)]$? ANSWER: ________________

11. Sketch the functions $f(x) = |x + 1| - 2$ and $g(x) = \sqrt{x - 4} + 3$, and annotate your graphs
13. Simplify \( f(x) = \frac{3x^9 - 2x}{2x^3} \). ANSWER: \( f(x) = \) 

14. Simplify \( \log_2(2x) \). ANSWER: 

15. Sketch the functions \( \log_2(2x) \) and \( 3^{-x} \), and annotate your graphs.

16. Simplify \( \log_{10}(e^x) \). ANSWER: 

17. Express the function \( f(x) = \ln \left( \frac{2x^3(x+1)}{(x-1)^2(x+2)^2} \right) \) as sums and differences of logarithms.

ANSWER: 

19. Solve the equation \( e^x = 2^{x+1} \). ANSWER: 

20. Write \( \left( \frac{1}{2} \right)^t \) as a natural exponential. ANSWER: 

Problem 2: Rational functions. [20 points] Consider the function $f(x) = \frac{x-2}{x-3}$

(a) What is the $x$–intercept?

(b) What is the $y$–intercept?

(c) What is the vertical asymptote?

(d) What is the horizontal asymptote?

(e) Draw a signs table for $f(x)$

(f) Using this information, sketch $f(x)$.

(g) Calculate the inverse of $f(x)$.

(h) Verify that $f \left[ f^{-1}(x) \right] = x$. 

Problem 3: Applied Problem [20 points]

A bank offers an interest rate of 3% annually. Supposing your initial investment is $10,000.

1. How much money will be in your bank account after 1 year? ________________

2. How much money will be in your bank account after 2 years? ________________
(no need to simplify or evaluate the answer)

3. The amount of money in your bank account \( A \) as a function of the number of years passed \( n \) is (circle the correct answer)

   - \( A(n) = 10,000(1.03)^{n+1} \)
   - \( A(n) = 10,000(0.03)^{n+1} \)
   - \( A(n) = 10,000(1.03)^n \)
   - \( A(n) = 10,000(0.03)^n \)

4. Graph \( A(n) \), making sure to annotate the important points (i.e. the \( y \)-intercept, and the answer to question 1).

5. Express the function \( A(n) \) as a natural exponential. ________________

6. How long would it take until you have $20,000 in the bank? Hint: you may need to use the fact that \( \ln(2) \approx 0.7, \ln(0.03) \approx -3.5, \ln(1.03) \approx 0.035 \).