Problem 1: Short questions [60 points]

In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than a 2 minutes per question. Each question is worth 3 points.

1. Find the linear function $f(x)$ such that $f(0) = 1$ and $f(1) = 0$. ?

2. What is the discriminant of the following quadratic function? $f(x) = bx^2 - cx + a$?

$D =$ ____________________________

Given the functions $f(x) = \frac{1}{x+1}$ and $g(x) = \sqrt{x-3}$

3. Write down, and then simplify the expression $f(x) - f(x-1)$.

4. What is the domain of $f(x)$? ____________________________

5. What is the domain of $g(x)$? ____________________________

6. What is the domain of $\frac{f}{g}(x)$ ? ____________________________
7. Sketch the function \( f(x) = -(x - 1)^3 + 2 \) and \( g(x) = \frac{1}{(x+1)^3} - 1 \)

Given the function \( f(x) = -x^2 - 4x - 2 \) and its graph:

9. Complete the square

10. What are the coordinates of the vertex? ________________________________

11. Does it open up or down? ________________________________

12. What is the equation of the tangent at the \( y \)-intercept? ________________________________

13. Does the function have roots? If so, what are they? Hint: \( \sqrt{2} \approx 1.4 \).

14. Based on this information, sketch the parabola \( y = -x^2 - 4x - 2 \), making sure to annotate your graph correctly.
15. Factor the quadratic function \( f(x) = 2x^2 + x - 1 \) using the quadratic formula.

16. Factor the following expression by grouping, and make sure your result is fully factored:
\( f(x) = x^4 + 2x^3 - 4x^2 - 8x \).

Given the function \( f(x) = \frac{x^2 - 1}{x + 0} \):

17. What is the name of this type of function? ________________________________

18. What are the \( x \)-intercepts? ________________________________

19. What is the vertical asymptote? ________________________________

20. What is the equation of the oblique asymptote? ________________________________
Problem 2. [20 points] Consider the function \( f(x) = x^4 - 4x^2 \).

(a) Behavior for large \( x \):
   - What is \( f(x) \) approximately equal to for large \(|x|\)?
   - When \( x \) tends to \(-\infty\), \( f(x) \) goes to
   - When \( x \) tends to \(+\infty\), \( f(x) \) goes to

(b) Is the function odd or even?

(c) Factor \( f(x) \)

(d) Determine the \( x \)- and \( y \)-intercepts
   - \( x \)-intercept(s):
   - \( y \)-intercept:

(e) Draw a signs table (make sure to include the zeros)

(f) Sketch the function \( f(x) \)
Problem 3. Applied Problem [20 points].

This problem guides you through the calculation of the optimal sales price for a new HP tablet. Market analysis shows that the likely number of tablet $N$ that would sell in the US every day is a linear function of the sales price, which we call $x$:

$$N(x) = -2x + 4000$$

**Question 1:** How many tablets would be sold (per day) if the price was $500 per tablet?

**Question 2:** Let the revenue $R$ be the amount of money made when selling $N$ tablets. What is the revenue (per day) as a function of the price $x$?

**Question 3:** What is the name of this type of function? ____________

**Question 4:** Factor $R(x)$ and draw its signs table. Use the table to sketch $R(x)$.

**Question 5:** What is the position of the maximum of $R(x)$?

**Question 6:** What is price HP should set their tablets at to make as much money as possible? ______________

**Question 7:** Above what price would they start losing money? ______________