In the first example of a second-order circuit we worked out in class, we came up with the following differential equation for *i(t)*:

$$\frac{d^2 i}{dt^2} + 6\frac{di}{dt} + 25i = 0$$

Using a trial solution of $i(t) = Ae^{\lambda t}$, the following characteristic equation is obtained:

$$\lambda^2 + 6\lambda + 25 = 0$$

Using the quadratic equation, we have the following solution:

$$\lambda = -3 \pm 4i$$

So, taking both solutions into account, *i(t)* becomes:

$$i(t) = Ae^{-3t + 4it} + Be^{-3t - 4it}$$

Factoring out the common e^{-3t} term and using Euler's identity, we end up with:

$$i(t) = e^{-3t}(Ae^{4it} + Be^{-4it}) = e^{-3t}(A\cos 4t + i\,A\sin 4t + B\cos 4t - i\,B\sin 4t) \qquad (1)$$

Applying the initial condition of $i(0) = 0$, we get the following equation:

$$i(0) = e^{0}(A\cos 0 + i\,A\sin 0 + B\cos 0 - i\,B\sin 0) = A + B = 0$$

So A = –B. Applying this result to equation (1) gives us:

$$i(t) = e^{-3t}(A\cos 4t + i\,A\sin 4t - A\cos 4t + i\,A\sin 4t) = e^{-3t}(2A\,i\sin 4t)$$

The goal, remember, was to find the voltage across the capacitor. Since $i(t) = C\frac{dv}{dt}$, we can find an expression for *v(t)*:

$$v(t) = \frac{1}{C}\int i(t)dt = \frac{1}{0.04}\int 2Aie^{-3t}\sin(4t)dt = 50Ai\int e^{-3t}\sin(4t)dt$$

To find the integral, we integrate by parts (recall: $\int u\ dv = uv - \int v\ du$)

$$\int e^{-3t}\sin(4t)dt = e^{-3t}\frac{-1}{4}\cos 4t - \int \frac{3}{4}e^{-3t}\cos(4t)dt$$

(let $u = e^{-3t}$ and $dv = \sin 4t\,dt$…then $du = -3e^{-3t}\,dt$ and $v = -(\cos 4t)/4$)

Integrating by parts one more time (for the integral on the right hand side) gives us:

$$\int e^{-3t}\sin(4t)dt = -\frac{1}{4}e^{-3t}\cos 4t - \frac{3}{4}\int e^{-3t}\cos(4t)dt$$

$$\int e^{-3t}\sin(4t)dt = -\frac{1}{4}e^{-3t}\cos 4t - \frac{3}{4}\left[\frac{1}{4}e^{-3t}\sin 4t + \int\frac{3}{4}e^{-3t}\sin(4t)dt\right]$$

(let u = e^{-3t} and dv = cos 4t dt…then du = $-3e^{-3t}$ dt and v = (sin 4t) / 4)

We can rearrange the equation and "solve" for the integral:

$$\int e^{-3t}\sin(4t)dt = -\frac{1}{4}e^{-3t}\cos 4t - \frac{3}{16}e^{-3t}\sin 4t - \frac{9}{16}\int e^{-3t}\sin(4t)dt$$

$$\frac{25}{16}\int e^{-3t}\sin(4t)dt = -\frac{1}{4}e^{-3t}\cos 4t - \frac{3}{16}e^{-3t}\sin 4t$$

$$\int e^{-3t}\sin(4t)dt = -\frac{4}{25}e^{-3t}\cos 4t - \frac{3}{25}e^{-3t}\sin 4t + K$$

where K is the integration constant (alternatively, you could look up this integral in a table)

So $v(t)$ can be written now as:

$$v(t) = 50Ai\left[-\frac{4}{25}e^{-3t}\cos 4t - \frac{3}{25}e^{-3t}\sin 4t + K\right]$$

$$v(t) = -2Aie^{-3t}[4\cos 4t + 3\sin 4t] + 50AiK$$

To find K, use the steady state value, i.e. $v(\infty) = 12$

$$v(t) = 12 = -2Aie^{-\infty}[4\cos\infty + 3\sin\infty] + 50AiK = -2Ai0[4\cos\infty + 3\sin\infty] + 50AiK$$

$$12 = 0 + 50AiK = 50AiK$$

So $\quad v(t) = -2Aie^{-3t}[4\cos 4t + 3\sin 4t] + 12$

To find A, use the other initial condition, i.e. $v(0) = 0$

$$v(0) = 0 = -2Aie^{0}[4\cos 0 + 3\sin 0] + 12 = -2Ai[4+0] + 12 = -8Ai + 12$$

$$0 = -8Ai + 12$$

$$8Ai = 12$$

$$A = \frac{12}{8i} = \frac{3}{2i}$$

So $$v(t) = -2\frac{3}{2i} ie^{-3t}[4\cos 4t + 3\sin 4t] + 12$$
$$v(t) = -3e^{-3t}[4\cos 4t + 3\sin 4t] + 12$$
$$v(t) = 12 - e^{-3t}[12\cos 4t + 9\sin 4t]$$

This is exactly the same result we got in class after re-solving the problem (by converting *i(t)* in the KCL equation to *v(t)* using the IV relationship for the capacitor). Obviously, this approach takes more time and effort, but you get the same result, as expected.