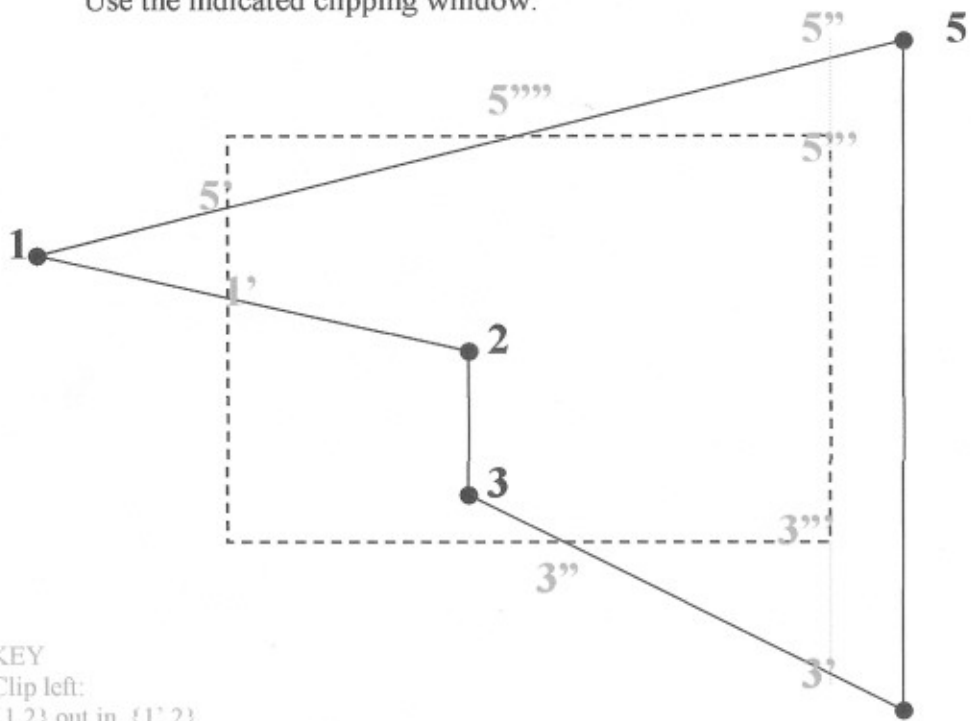


Apply the Sutherland-Hodgeman clipping algorithm to the polygon (vertices 1-5).  
Use the indicated clipping window.



#### KEY

Clip left:

- {1,2} out in {1',2}
- {2,3} in in {3}
- {3,4} in in {4}
- {4,5} in in {5}
- {5,1} in out {5'}

Clip Right

- {1',2} in in {2}
- {2,3} in in {3}
- {3,4} in out {3'}
- {4,5} out out {}
- {5,5'} out in {5'', 5'}
- {5',1'} in in {1'}

Clip Bottom

- {2,3} in in {3}
- {3,3'} in out {3''}
- {3',5'} out in {3'', 5''}
- {5'', 5'} in in {5'}
- {5',1'} in in {1'}
- {1',2} in in {2}

Clip Top

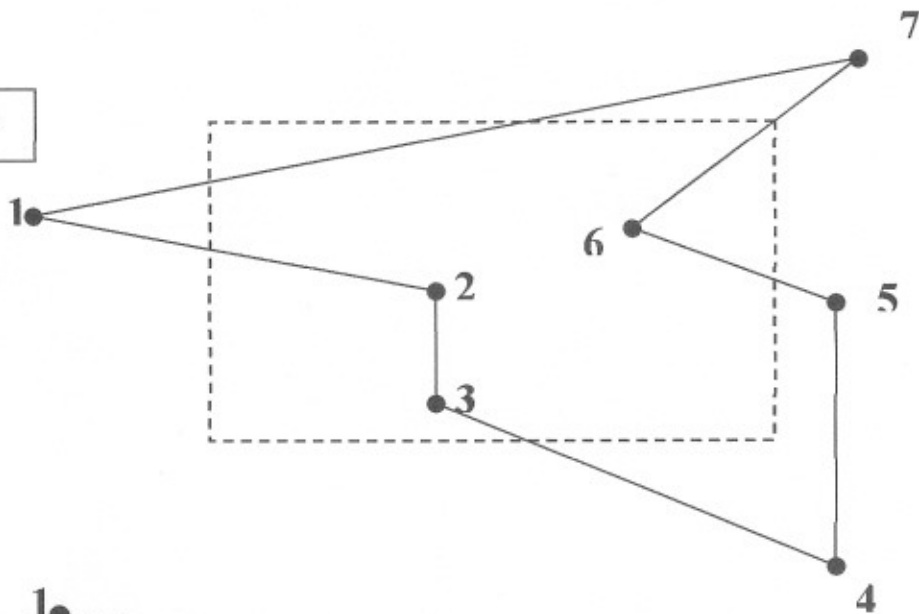
- {3,3''} in in {3''}
- {3'', 3'''} in in {3'''}
- {3''', 5''} in out {5'''}
- {5''', 5'} out in {5''', 5'}
- {5', 1'} in in {1'}
- {1',2} in in {2}
- {2,3} in in {3}

2. Using the indicated clipping window, is it possible to apply the Sutherland-Hodgeman clipping algorithm to the following polygons? Answer Yes or No for both 2a and 2b.

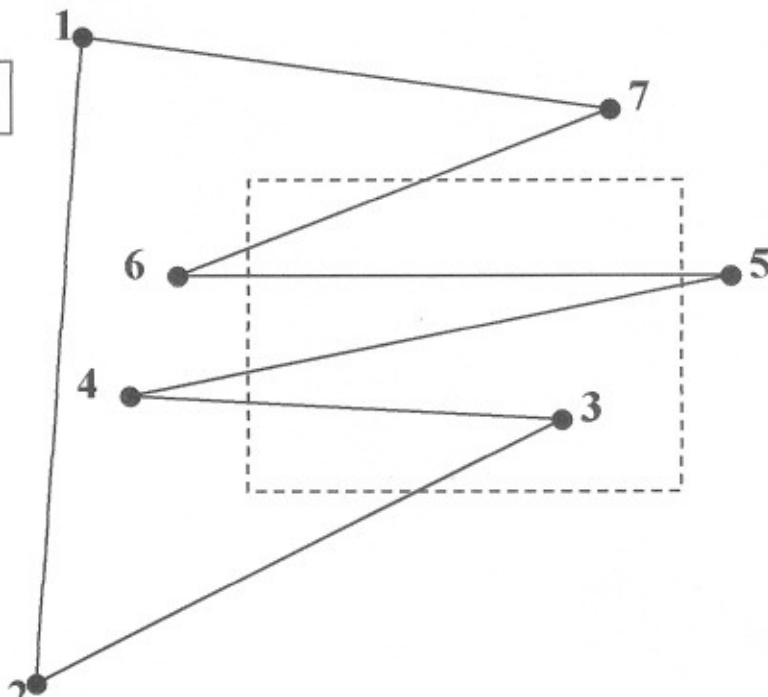
If your answer is yes state why it is possible,  
If your answer is no, state why it is not possible  
(1-2 sentences should be fine).

- a) Yes only a single set of vertices will be left  
b) no more than a single set of vertices will be left

2A

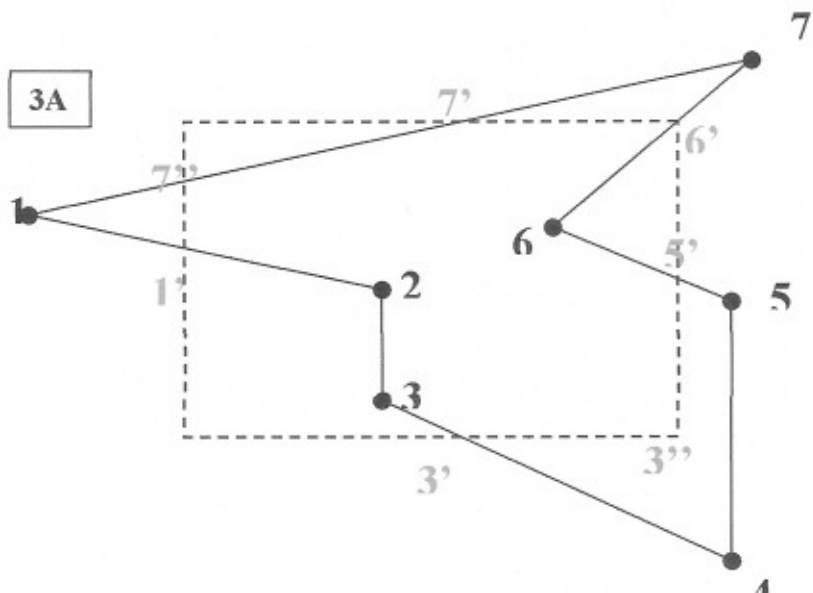


2B



3. Apply the Weiler-Atherton clipping algorithm to the polygons below (vertices 1-7). Use the indicated clipping window. Lightly shade the result polygons. Show all work!

Shortest acceptable Answer (Not actually 100% correct, but close enough to understand how the algorithm works):  $1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 3' \Rightarrow 3'' \Rightarrow 5' \Rightarrow 6' \Rightarrow 7' \Rightarrow 7'' \Rightarrow 1' \Rightarrow 2$   
 Note that I ended on a previously processed point



4. Generate the binary ray tracing trees for pixels 1 and 2 given the scene below. I have indicated the first reflection and transmission vectors for pixel 1. You should draw in subsequent reflection and transmission vectors if they exist. Use these vectors to generate your binary ray tracing trees.

Told that all normals for emitters (if needed) are



L1 and L2 are non-reflecting emitters

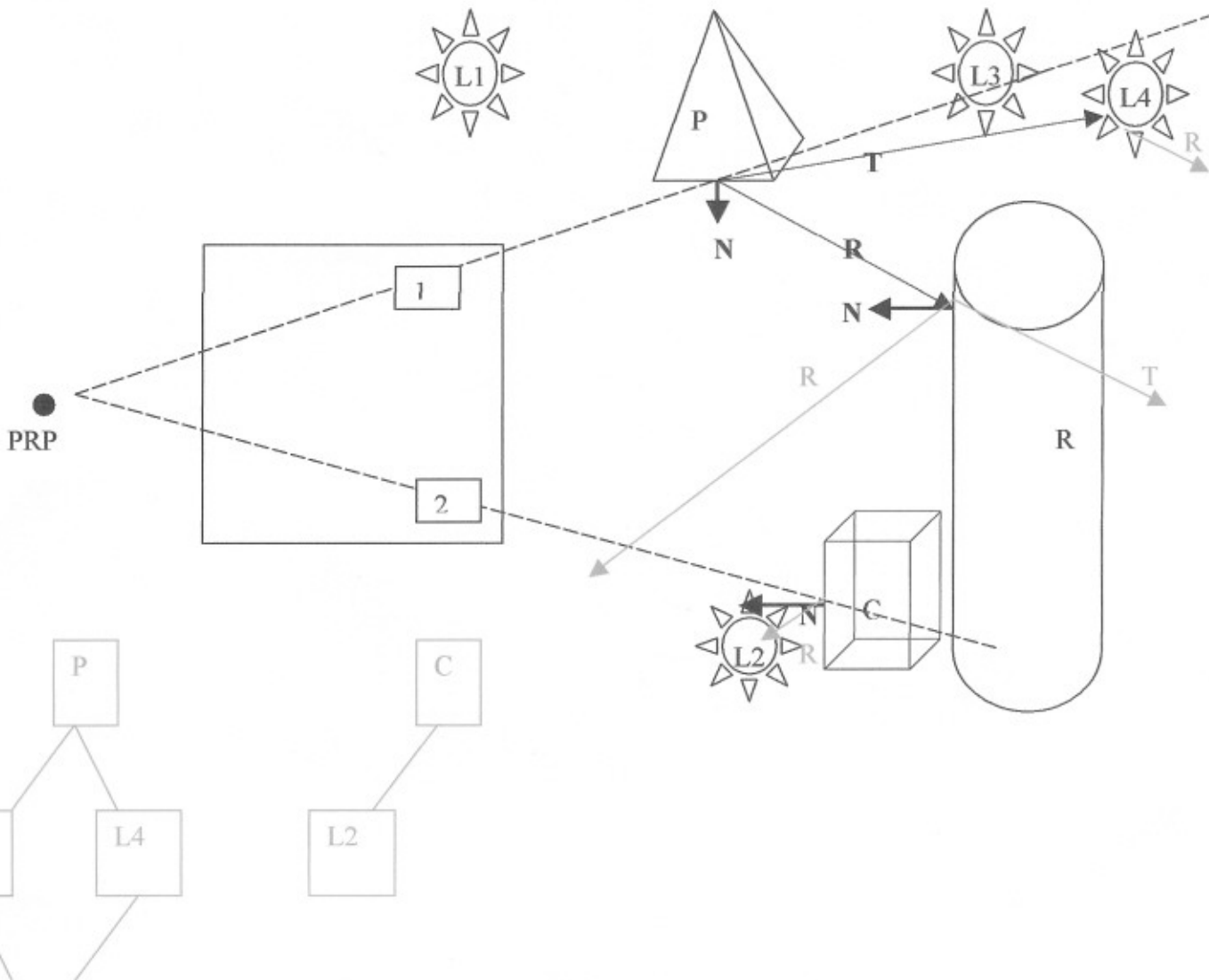
L3 and L4 are reflecting emitters

C, the cube, is completely opaque

R, the round cylinder, is semitransparent

Angle of refraction is not important as long as the refraction ray, is shown continuing through the cylinder (It could never bounce forward of the cylinder).

P, the pyramid, is semitransparent



5. a. What is the transmission vector for the semi-transparent pyramid?  
 b. What is the angle of reflection? (What is the value of  $\theta_x$ ?)  
 c. What is the angle of refraction?

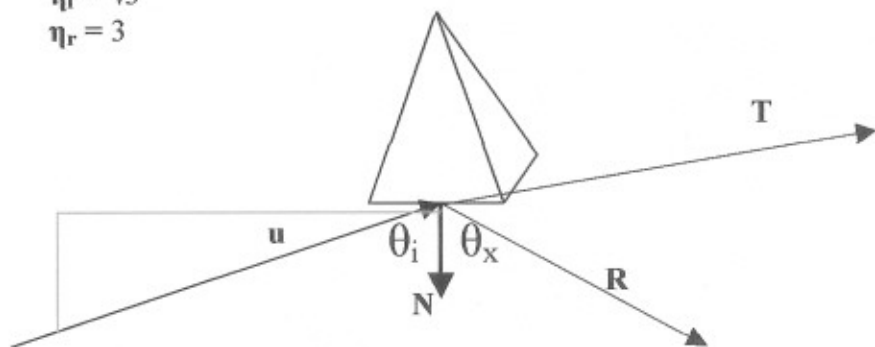
$$\mathbf{N} = (0, -1, 0)$$

$$\theta_i = 60^\circ$$

$$\mathbf{u} = (\sqrt{3}/2, 1/2, 0)$$

$$\eta_i = \sqrt{3}$$

$$\eta_r = 3$$



$$\text{KEY: } \mathbf{T} = \eta_i/\eta_r \mathbf{u} - (\cos \theta_r - \eta_i/\eta_r \cos \theta_i) \mathbf{N}$$

$$\cos \theta_r = \sqrt{1 - (\eta_i/\eta_r)^2 (1 - \cos^2 \theta_i)}$$

$$\begin{aligned} \cos \theta_r &= \sqrt{1 - (\sqrt{3}/3)^2 (1 - \cos^2 60^\circ)} = \sqrt{1 - (\sqrt{3}/3)^2 (1 - (1/2)^2)} \\ &= \sqrt{1 - (3/9) (1 - (1/4))} = \sqrt{1 - (3/9) (3/4)} = \sqrt{1 - 1/4} = \\ &= \sqrt{3/4} = \sqrt{3}/2 \end{aligned}$$

$$\theta_r = \text{the angle of refraction} = 30^\circ$$

$$\mathbf{T} = \eta_i/\eta_r \mathbf{u} - (\cos \theta_r - \eta_i/\eta_r \cos \theta_i) \mathbf{N}$$

$$\mathbf{T} = \sqrt{3}/3 \mathbf{u} - (\sqrt{3}/2 - (\sqrt{3}/3) (1/2)) \mathbf{N} = \sqrt{3}/3 \mathbf{u} - (\sqrt{3}/2 - \sqrt{3}/6) \mathbf{N}$$

$$\mathbf{T} = \sqrt{3}/3 \mathbf{u} - (\sqrt{3}/3) \mathbf{N}$$

$$\mathbf{T} = \sqrt{3}/3 (\sqrt{3}/2, 1/2, 0) - (\sqrt{3}/3) (0, -1, 0)$$

$$\mathbf{T} = (3/6, \sqrt{3}/6, 0) - (0, -\sqrt{3}/3, 0)$$

$$\mathbf{T} = (3/6, \sqrt{3}/6, 0) - (0, -\sqrt{3}/3, 0) = (3/6, \sqrt{3}/6, 0) + (0, \sqrt{3}/3, 0)$$

$$\mathbf{T} = (1/2, \sqrt{3}/2, 0)$$

6. Use Gouraud shading to determine the color at point P. The normals for each of the patches are indicated. (NOTE: This is the same vertex and normal set found in the Phong shading problem.) You may leave all answers in fractional form.

R, G, B

$$V_1 I_e(1, 1, 0.5)$$

$$V_2 I_e(0.5, 1, 0.5)$$

$$V_3 I_e(1, 0.5, 0.5)$$

$$V_4 I_e(1, 0.5, 0.5)$$

$$\text{Light Vector } L = 0, 1, 1$$

$$I_a = 2/3 \text{ (monochromatic)}$$

$$k_a = 3/4$$

$$k_d = \sqrt{2}/2$$

$$k_s = 0$$

KEY:

Light vector is  $(0, \sqrt{2}/2, \sqrt{2}/2)$

Normals:

$$V_1 = 0, 0, 1 \quad V_2 = 0, 1, 0$$

$$V_3 = 1, 0, 0 \quad V_4 = 1, 0, 0$$

$$k_a I_a = 2/3 * 3/4 = 1/2 = 0.5$$

$$V_1 = k_a I_a + k_d I_e (N \cdot L)$$

$$(N \cdot L) = (0, 0, 1) \cdot (0, \sqrt{2}/2, \sqrt{2}/2) = \sqrt{2}/2$$

$$k_d I_e (N \cdot L) = k_d (N \cdot L) I_e = \sqrt{2}/2 (\sqrt{2}/2) I_e$$

$$= 1/2 (I_e) = (.5, .5, .25)$$

$$k_a I_a = .5$$

$$= (.5 + .5, .5 + .5, .5 + .25) = (1, 1, .75)$$

$$V_2 = k_a I_a + k_d I_e (N \cdot L)$$

$$(N \cdot L) = (0, 1, 0) \cdot (0, \sqrt{2}/2, \sqrt{2}/2) = \sqrt{2}/2$$

$$k_d I_e (N \cdot L) = \sqrt{2}/2 (\sqrt{2}/2) I_e$$

$$= 1/2 (I_e) = (.25, .5, .25)$$

$$= (.5 + .25, .5 + .5, .5 + .25) = (.75, 1, .75)$$

$$V_3 \& V_4 = k_a I_a + k_d I_e (N \cdot L)$$

$$(N \cdot L) = 1, 0, 0 \cdot (0, \sqrt{2}/2, \sqrt{2}/2) = 0$$

$$k_a I_a = (.5, .5, .5)$$

$$I_1 = 1/2(V_3) + 1/2(V_1) = (.25, .25, .25) + (.5, .5, 3/8) = (3/4, 3/4, 5/8) = (.75, .75, .625)$$

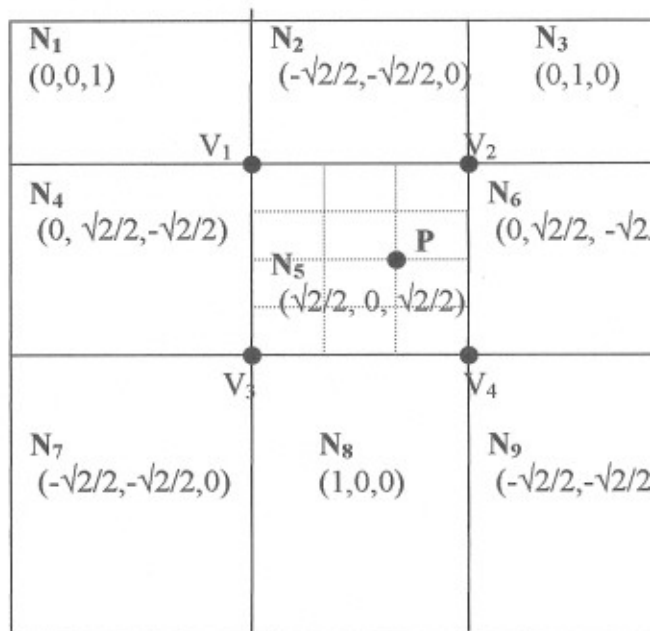
$$I_2 = 1/2(V_2) + 1/2(V_4) = (3/8, 1/2, 3/8) + (1/4, 1/4, 1/4) = (5/8, 3/4, 5/8) = (.625, .75, .625)$$

$$P = 2/3(I_2) + 1/3(I_1) = 2/3(5/8, 3/4, 5/8) + 1/3(3/4, 3/4, 5/8)$$

$$= (10/24, 6/12, 10/24) + (3/12, 3/12, 5/24)$$

$$= (5/12, 6/12, 5/12) + (3/12, 3/12, 5/24) = (8/12, 9/12, 15/24) = (2/3, 3/4, 5/8)$$

$$= (.67, .75, .625)$$



7. Use Phong shading to determine the color at point P. (NOTE: This is the same vertex and normal set found in the Gouraud shading problem.) You may leave all answers in fractional form.

Light Vector  $L = 0, 1, 1$

Viewing vector  $V = 1, -1, -1$

$I_l = 1$  (Monochromatic light)

$k_a = 0$        $k_d = 0$        $k_s = 18/98$

$\eta_s = 2$

KEY:

Light vector is  $(0, \sqrt{2}/2, \sqrt{2}/2)$

Viewing vector is  $(\sqrt{2}/2, 0, -\sqrt{2}/2)$

Normals:

$V1=0,0,1$        $V2=0,1,0$

$V3=1,0,0$        $V4=1,0,0$

Normal A =  $1/2(\text{Normal } V1 + \text{Normal } V3)$

$$= 1/2(1, 0, 1) = (1/2, 0, 1/2)$$

$$= (\sqrt{2}/2, 0, \sqrt{2}/2)$$

Normal B =  $1/2(\text{Normal } V2 + \text{Normal } V3)$

$$= 1/2(0, 1, 0 + 1, 0, 0) = (1/2, 1/2, 0)$$

$$= (\sqrt{2}/2, \sqrt{2}/2, 0)$$

Normal at P =  $1/3(\text{Normal } A) + 2/3(\text{Normal } B)$

$$= 1/3(\sqrt{2}/2, 0, \sqrt{2}/2) + 2/3(\sqrt{2}/2, \sqrt{2}/2, 0)$$

$$= (\sqrt{2}/6, 0, \sqrt{2}/6) + (2\sqrt{2}/6, 2\sqrt{2}/6, 0)$$

$$= (3\sqrt{2}/6, 2\sqrt{2}/6, \sqrt{2}/6)$$

$$= (\sqrt{2}/2, \sqrt{2}/3, \sqrt{2}/6)$$

$$\text{Magnitude} = \sqrt{(2/4 + 2/9 + 2/36)} = \sqrt{18/36 + 8/36 + 2/36} = \sqrt{28/36} = \sqrt{7/9} = \sqrt{7}/3$$

$$\text{Normalized Normal at P} = (3/\sqrt{7}) \cdot ((\sqrt{2}/2, \sqrt{2}/3, \sqrt{2}/6)) = (3\sqrt{2}/2\sqrt{7}, \sqrt{2}/\sqrt{7}, \sqrt{2}/2\sqrt{7})$$

Find the halfway vector  $H = L + V / |L + V|$

$$L + V = 1, 0, 0$$

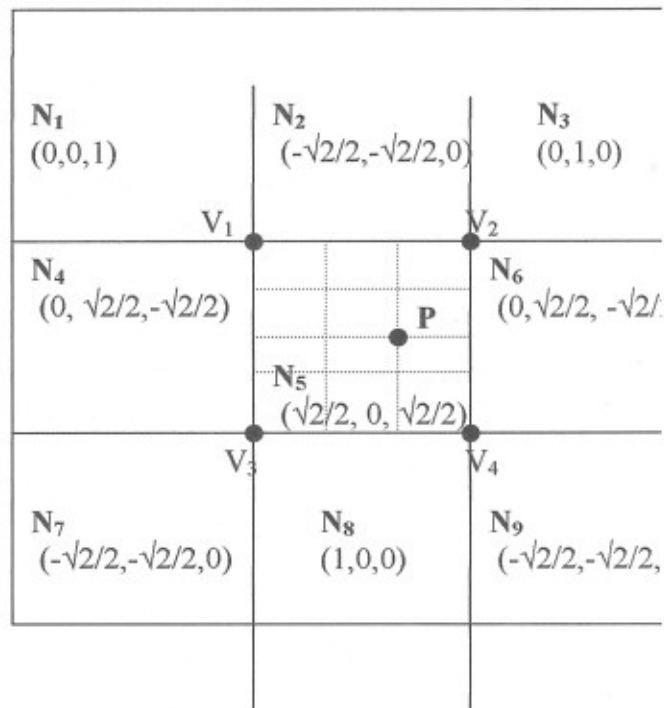
$$|L + V| = 1$$

$$H = (1, 0, 0)$$

$$N \cdot H = 3\sqrt{2}/2\sqrt{7}$$

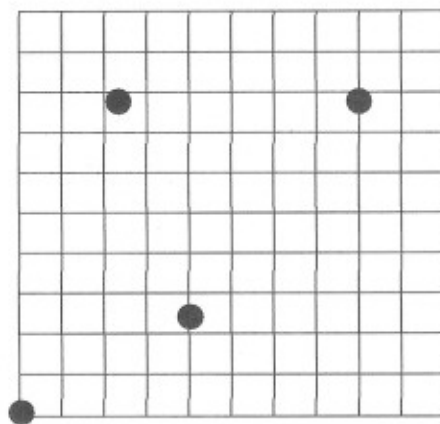
Light is now  $I = I_l k_s (N \cdot H)^{\eta_s}$

$$I = 1 * 18/98 * (18 * 18 / 28 * 28) = 5832 / 76832 = .0759 \text{ (monochromatic light)}$$



8. We wish to approximate a parametric curve using a Bezier spline. We are given four control points:  $(0,0,0)$ ,  $(24,0,80)$ ,  $(40,0,24)$ ,  $(80,0,80)$ .

- What is the coordinate position for the parametric value  $u=0.5$ .
- Indicate the coordinate you calculated on the graph. Draw the curve (I just want a rough approximation)



KEY (Bezier interpolates point 1 and point 4 (drawing is a little off, best I could do on my computer)

$$P(u) = (1-t)^3 P_1 + 3t(1-t)^2 P_2 + 3t^2(1-t) P_3 + t^3 P_4$$

$$P(.5) = (1-.5)^3 P_1 + 3(.5)(1-.5)^2 P_2 + 3(.5)^2(1-.5) P_3 + (.5)^3 P_4$$

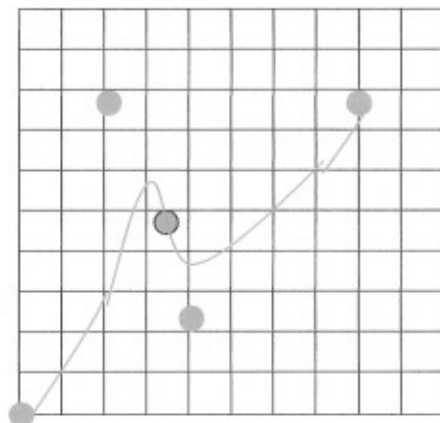
$$P(.5) = (1/2)^3 P_1 + 3(1/2)(1/2)^2 P_2 + 3(1/2)^2(1/2) P_3 + (1/2)^3 P_4$$

$$(1/2)^3 = (1/8)$$

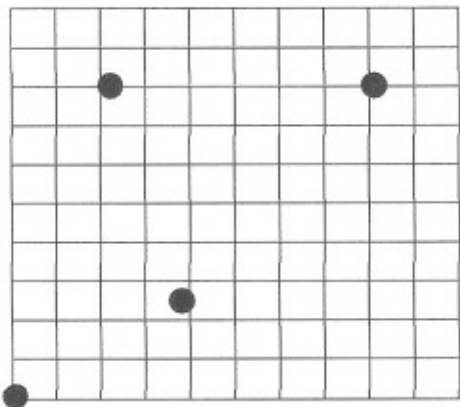
$$P(.5) = 1/8(0,0,0) + 3/8(24,0,80) + 3/8(40,0,24) + 1/8(80,0,80)$$

$$= (0,0,0) + (9,0,30) + (15,0,9) + (10,0,10)$$

$$= (34,0,49)$$



9. We wish to interpolate a parametric curve using a Cardinal spline. We are given four control points:  $(0,0,0)$ ,  $(24,0,80)$ ,  $(40,0,24)$ ,  $(80,0,80)$ . Let tension =  $-3$ .
- What is the coordinate position for the parametric value  $u=0.5$ .
  - Indicate the coordinate you calculated on the graph. Draw the curve (I just want a rough approximation)



KEY

a.  $s = (1-t)/2 = (1-3)/2 = 2$

$$u_{term\_p1} = (-s^3 u^3) + (2s^2 u^2) - (s^2 u);$$

$$= -2(1/2)(1/2)(1/2) + 2(2)(1/2)(1/2) - (2)(1/2) = -1/4 + 1 - 1 = -1/4$$

$$u_{term\_p2} = ((2.0-s)^3 u^3) + ((s-3.0)^2 u^2) + 1.0;$$

$$= 2 - 2(1/2)(1/2)(1/2) + (2-3)(1/2)(1/2) + 1 = -1/4 + 1 = 3/4$$

$$u_{term\_p3} = (s-2.0)^3 u^3 + (3.0-2.0s)^2 u^2 + s^2 u;$$

$$= 0(1/2)(1/2)(1/2) + ((3-4))^2 (1/2)(1/2) + (2)(1/2) = 0 - 1/4 + 1 = 3/4$$

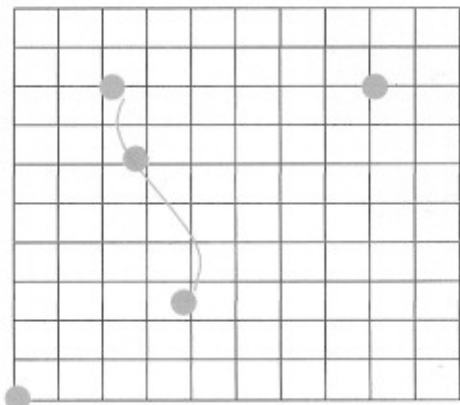
$$u_{term\_p4} = s^3 u^3 - s^2 u^2;$$

$$= 2^3 (1/2)^3 - 2^2 (1/2)^2 = 1/4 - 1/2 = -1/4$$

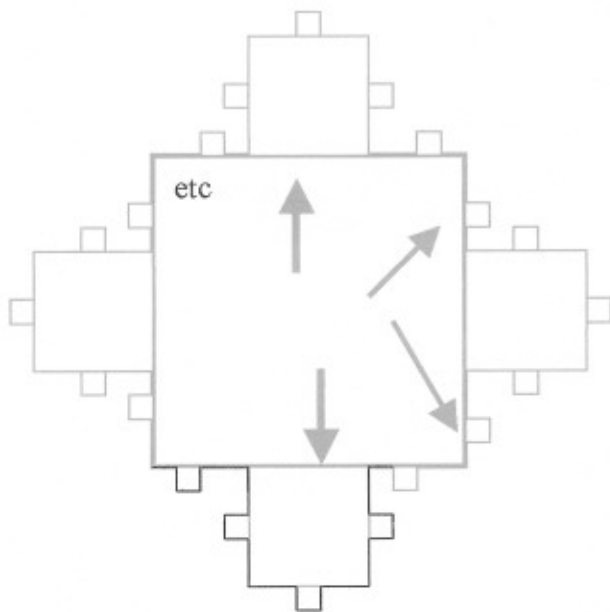
$$P(u) = -1/4 P_1 + 3/4 P_2 + 3/4 P_3 - 1/4 P_4$$

$$= -1/4(0,0,0) + 3/4(24,0,80) + 3/4(40,0,24) - 1/4(80,0,80)$$

$$= (0+18+30-20), (0), (0+60+18-20) = (28, 0, 58)$$



10. We wish to generate a fractal. We are given an initiator and a generator.
- Draw the second iteration of the fractal. (The zero<sup>th</sup> iteration is the initiator)
  - Determine the fractal dimension of the following deterministic fractal. (Leave it as a ratio of natural logs.) Note: Lines connecting the fractals are not really part of the fractal (See arrows)



There are 5 new segments generated from each old segment.  
Each old segment is divided into 3 parts.

$$D = \ln(5) / \ln(3)$$

11. We are given the the following grammar to generate a tree.

The tree alphabet is  $\{A, B, [, ], (, )\}$

The rules to generate **words** are as follows:

$A \Rightarrow AA[B]A$

$B \Rightarrow AA(B)A[B]A$

The starting basis ( $tree_0$ ) is B.

[ Starts a left branch, ] Ends the right branch

( Starts a right branch, ) Ends the right branch.

a. Show me the resulting WORD which will generate  $tree_2$ .

b. Now I want to draw  $tree_2$ . Of course grammars have no inherent geometry.

I will need to introduce a branch angle. Branch angle= $45^\circ$

KEY:

Tree0 = B

Tree1 = AA(B)A[B]A

Tree2 = AA[B]A -AA[B]A-(AA(B)A[B]A) -AA[B]A-[AA(B)A[B]A] -AA[B]A

