CMP 132 Handout #6
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Final Review

This is a review sheet/study guide for the final exam.
There will be a review session Tuesday 1:00–2:30 in BE 360.
The final exam will be Wednesday, March 19 from 12:00 to 3:00 P.M.

Before the midterm we covered a variety of topics. See the *midterm review sheet* for details. Any of the material there (and in the associated chapters) may be on the final.

After the midterm we covered the second half of chapter 12 through 15 in the 2nd edition of the book (chapter 11 through 14 in the 3rd edition). Some topics (especially the chapter on computable functions) were covered in less detail. The particular topics include:

1. Encodings and the relationships between decision problems and languages, decidable and undecidable problems/languages.
2. Reductions between problems: computable reduction functions, if-and-only-if property, $L_1 \leq L_2$ notation.
3. Halting problem.
4. Many examples of reductions and undecidable problems.
5. Rice's Theorem.
6. Post's Correspondence Problem (PCP), Halting reduces to MPCP which reduces to PCP. This involves encoding halting computations of a TM with correspondences of an MPCP instance.
7. $\star$ Undecidable grammar problems: some reductions from PCP, halting computations as intersections of 2 CF languages, etc.
8. Computable and Uncomputable functions, Busy beaver function.
9. $\star$ Initial functions, primative recursive functions, bounded quantification is primative recursive.
10. $\star$ Bounded minimialization, unbounded minimalization and the μ-recursive functions.
11. $\star$ Gödel numbering and encoding of TM configurations as a number.
12. Computational complexity theory, time and space resources for TMs and non-deterministic TMs as a function of input size.
13. Complexity classes: Time($f(n)$), Space($f(n)$), NTime($f(n)$), NSpace($f(n)$).
14. "Normal" or "nice" $f(n)$: the step counting functions.
15. Speedup theorems (we emphasized time, but Space, NTime, NSpace, also have speedup theorems).
16. Diagonalization and hierarchy Theorems: Time($f(n)$) is smaller than Time($Cn^2 f(n)^2$).

17. Containments between complexity classes, e.g. Time($f(n)$) is contained in Space($f(n)$), Space($f(n)$) is contained in Time($C^{f(n)}$), etc.

18. Optimization problems, related decision problems, "reasonable" encodings, and languages. Using a subroutine for the decision problem to solve the related Optimization problem.

19. "Tractable" problems, definitions of $\mathcal{P}$ and $\mathcal{NP}$, closure and robustness properties of $\mathcal{P}$, showing that a problem is in $\mathcal{NP}$.

20. Polynomial time reductions, and their properties and consequences (for example, $L_1 \leq_P L_2$ and $L_2 \in \mathcal{P}$ imples $L_1 \in \mathcal{P}$).

21. Definition of $\mathcal{NP}$-completeness. Why showing that a problem is $\mathcal{NP}$-complete is "evidence" (but *not* a proof) that the problem is not in $\mathcal{P}$.

22. CNF-satisfiability is $\mathcal{NP}$-complete (Cook's Theorem).

23. Basic $\mathcal{NP}$-complete problems and the reductions showing that they are $\mathcal{NP}$-complete: 3-Sat, Clique, Vertex Cover, k-colorability, subset sum.

24. $\star$ Co-$\mathcal{NP}$ and the possible "states of the world." Borderline problems: Primality and Graph Isomorphism.

25. $\star$ Dealing with $\mathcal{NP}$-completeness: Heuristics and approximation algorithms (if time permits).

Those sections marked with a $\star$ were covered quickly. Although they may appear on the final in short-answer or true-false questions, there will not be a long (i.e. proof) question on those topics. The exam will be comprehensive, and I expect that 20–35% of the points will be on material covered before the midterm.