FINAL

CIS 130 - Fall 08

Manfred Warmuth

NAME: Sols

Student ID:__________

This exam is closed book and closed notes.

Show partial solutions to get partial credit.

If your questions are not written legibly, you won't get full credit.

Clarity and succinctness will be rewarded!

question 1:__________(out of 20)

question 2:__________(out of 10)

question 3:__________(out of 10)

question 4:__________(out of 10)

question 5:__________(out of 10)

question 6:__________(out of 10)

question 7:__________(out of 10)

question 8:__________(out of 10)

question 9:__________(out of 10)

Total:__________(out of 100)

extra credit:__________(out of 10)

1. **a)** List a closure property of deterministic context free languages that does not hold for general context free languages

complement

b) What is the Church-Turing Thesis?

TMs are the most powerful
computational device

c) Give a language that is context free but not regular, and a second language that is not context free.

$\{a^n b^n : n \geqslant 0\}$ $\{a^n b^n c^n : n \geqslant 0\}$

d) Give a description of the grammar class that generates the regular languages.

$A \rightarrow cB$

$C \rightarrow D$

$Q \rightarrow r$

$T \rightarrow \Lambda$

right linear
grammars

e) Let L be the language containing only the single word w, where

$$w := \begin{cases} 0 & \text{if God does not exist} \\ 1 & \text{if God does exist.} \end{cases}$$

Is L decidable? Why or why not?
(Note that the answer does not depend on your religious convictions.)

L is either $\{0\}$ or $\{1\}$

Either languages is regular / decidable

f) Give a language that is not Turing acceptable.

NSA = { e(T) : T is TM that does not accept e(T) }

g) Are Turing acceptable languages closed under infinite union? Prove your answer.

NO! NSA = $\{w_1, w_2, \dots\}$

$L_i := \{w_i\}$ regular & T.a.

But $\bigcup_{i=1}^{\infty} L_i$ = NSA not T.a.

f) What does the universal Turing machine do?

Upon input e(T) e(z), where T is TM and z is input word,
it simulates T on input z

e) What is the language associated with the Halting Problem?

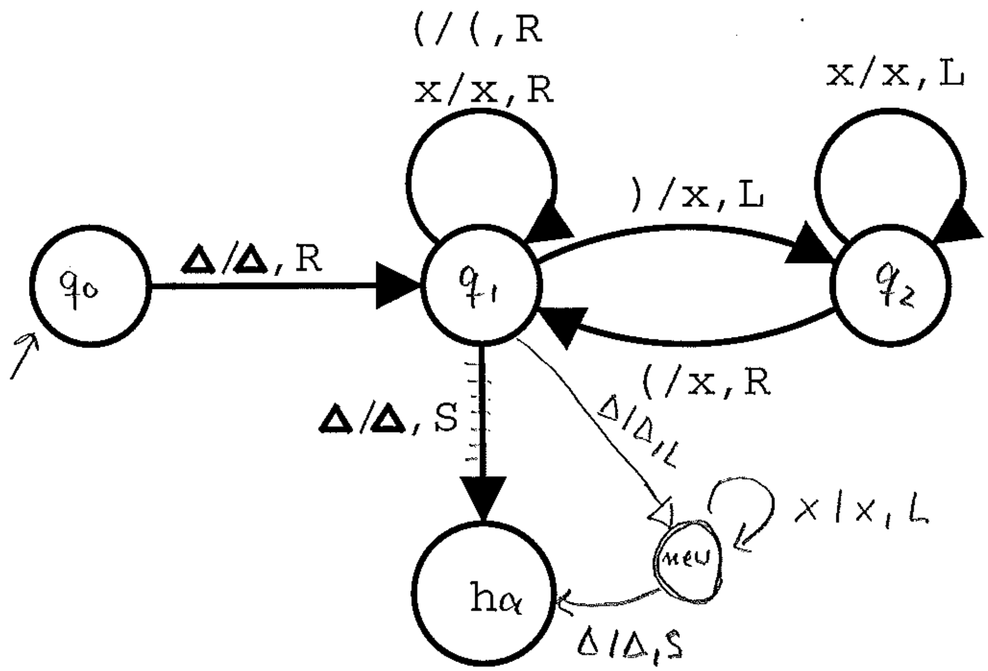
H = { e(T) e(z) : T is TM
z is input word
T halts on input z }

2. Is the following Turing machine accepting the words $(()())$ and $()(()$?

Δ ~~(()())~~ Δ $\qquad\qquad$ Δ ~~()~~ (~~()~~ Δ

yes $\qquad\qquad$ yes

In general, a (deterministic) Turing Machine accepts a word w, if when started on the left end of the tape $\Delta, w_1, \ldots, w_{|w|}, \Delta, \Delta, \ldots$ from the start state, then the machine runs into the accept state h_a.



Goes right to find)
Scan left to match
w. properly nested)
Stops when no more)
is found.

What is the language over the alphabet $\{(,)\}$ accepted by this Turing machine **and** how is it related to the language of balanced parens?

L

It accepts Prefix(L)

Fix the machine so that it accepts the language of balanced parens.

Hint: Essentially you need to add one state.

The new state checks whether all parens were wiped out.

3. Can the following grammar

$$
\begin{aligned}
S &\rightarrow AB|AC|BC\\
A &\rightarrow AA|AB|a\\
B &\rightarrow BB|BC|b\\
C &\rightarrow CC|AC|a
\end{aligned}
$$

generate the string *baab*?

Use the CYK algorithm!

Show your work by giving the table!

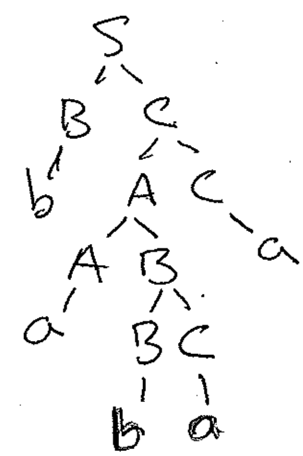
How can the table be used to answer the question whether the string can be generated or not?

b	a	b	a	a
B	A,C	B	A,C	A,C
S,B	S,A	S,B	A,S,C	
B	S,A,C	S,B		
S,B	S,A,C			
$\boxed{S}$,B				

Yes the String can be generated because we have an S in the final box.

one generation would be

S → BC → bAC → bABa → baBCa → babaa



4. Convert the following grammar into Chomsky Normal Form. Show your steps! The terminal letters are a and b.

$$
\begin{aligned}
S &\to AAS|BA \\
A &\to aA|\lambda \\
B &\to Sb|bA|\lambda
\end{aligned}
$$

S, A, B nullable

$S \to AAS|AA|AS|BA|A|B$

$A \to aA|a$

$B \to Sb|bA|b$

S chain rules

$$
\begin{tikzcd}
 & S \arrow[dl] \arrow[dr] & \\
A & & B
\end{tikzcd}
$$

$S \to AAS|AA|AS|BA|aA|a|Sb|bA|b$

$A \to aA|a$

$B \to Sb|bA|b$

$S \to AQ|AA|AS|BA|T_aA|a|ST_b|T_bA|b$

$Q \to AS$

$A \to T_aA|a$

$B \to ST_b|T_bA|b$

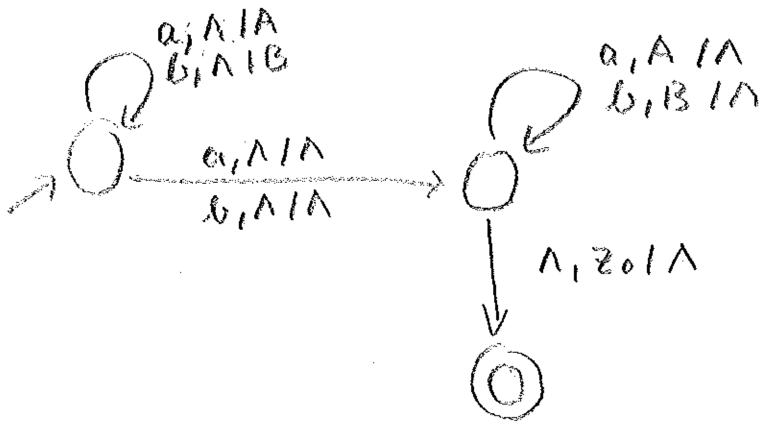
$T_a \to a$

$T_b \to b$

5. Construct a PDA accepting all words that are palindromes of odd length. More precisely the PDA is to accept the language

$$L = \{w \in \{a,b\}^* : |w| \text{ is odd and } w = w^R\}.$$

Check your PDA on small words!
State your acceptance criterion!



Either acceptance criterion

6. Reason that for any context-free language $L \subseteq \Sigma^*$ (where Σ is a finite alphabet), the following language is context-free as well:

$$\hat{L} = \{x \in L : |x| \geq 10\}.$$

Hint: What is the language of all words over the alphabet Σ of length at least 10?
Use a closure property!

$$\hat{L} = \underbrace{L \cap \underbrace{\overline{\bigcup_{i=0}^{10} \Sigma^i}}_{\text{regular}}}_{\text{context free}}$$

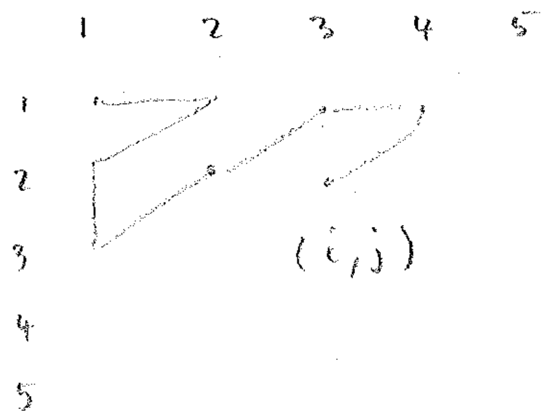
regular
because Σ^i regular and
regular langs closed
under $\cup$ and $\overline{}$

context free because
" langs closed
under $\cap$ w. regular set

7. Let S be the set of all triplets of natural numbers, i.e. $S = \{(i, j, k) | i, j, k \in \mathbb{N}\}$.
Show that S is countably infinite.

Hint: First show that the set of all pairs is countably infinite.

I $\quad \mathbb{N} \times \mathbb{N}$ c.i.



II $\quad S_n := \{(i,j,k) : i+j+k = n\}$

↑ finite

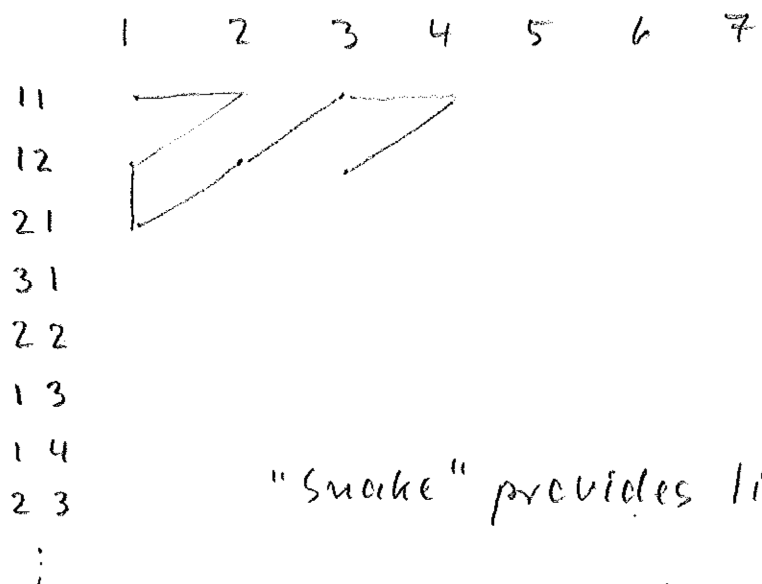
$S = \bigcup_{i=3}^{\infty} S_n$

So S is c.i. union of countable sets and is therefore c.i.

The "snake" provides a list

$\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ c.i.

because its equal to $(\mathbb{N} \times \mathbb{N}) \times \mathbb{N}$



Above list of $\mathbb{N} \times \mathbb{N}$

"Snake" provides list for

$(\mathbb{N} \times \mathbb{N}) \times \mathbb{N}$

8. Show that the set of all subsets S of the natural numbers $\mathbb{N}$ is uncountably infinite.

Hint: Use a diagonalization argument.

Assume S countably infinite. Then ... S listable

List of S

$$\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
S_1 & 0 & 1 & 1 & 0 & \\
S_2 & 1 & 1 & 0 & 1 & \\
S_3 & 0 & 1 & 0 & 1 & \\
S_4 & 1 & 1 & 1 & 0 & \\
\vdots & & & & &
\end{array}$$

$$T(i,j) = \begin{cases} 1 & \text{if } j \in S_i \\ 0 & \quad \notin \end{cases}$$

Construct set $D = \{q : q \notin S_q\}$
corresponding to complement of diagonal

$D \subseteq \mathbb{N}$

By assumption $\exists p : D = S_p$

But D can't be on the list since
it differs from all sets on the list

Either $p \in D$ or $p \notin D$:

Case $p \in D = S_p$. Then by construction of D, $p \notin D$

Case $p \notin D = S_p$.   " $\in$

4

T_H

9. Show that if the Halting Problem was decidable then Collatz's Conjecture could be resolved.

   Lothar Otto Collatz conjectured that for any integer n, the iterated application of the following function always converges to 1.

$$c(n) = \begin{cases} n/2 & \text{if } n \text{ even} \\ 3n+1 & \text{if } n \text{ odd.} \end{cases}$$

   Example sequence: 7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 1

   The conjecture has been checked for all $n \leq 2\,10^{15}$.

   Hint: First use the Halting Problem to decide whether a particular integer n converges to 1. Now use this in a loop and apply the Halting Problem again.

   For your construction may assume the existence of a TM T_c that computes function c.

By iteratively applying T_C construct a machine T that does the following:

T halts on input n iff iterate applications of c converge to 1 on n.

Now construct a new machine T' that halts iff $\exists n$ s.t. n does not converge to 1.

T': ignore input
for $n = 2$ step 1 do
if T_H decides no on $e(T)\,e(n)$ then halt

T_H decides no on $e(T')\,e(1)$ iff conjecture is true

10. (Extra Credit): Prove that the Halting Problem is undecidable.

Three proofs in
second to last class

1) If H decidable then SA decidable
2) Direct proof by diagonalization
mimicing proof given in first
class at TM level
3) Turing's proof via the computable
reals