MID TERM

CIS 130 - Fall 08

Warmuth

NAME: Sols

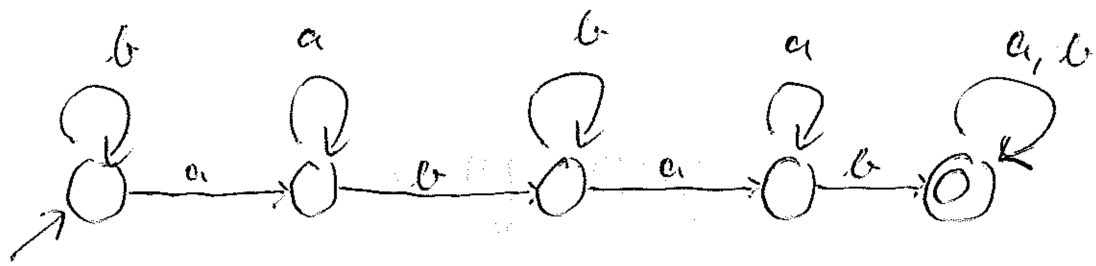
Student ID:

This exam is closed book and notes. Show partial solutions to get partial credit.
If your answers are not written legibly, you won't get full credit.
Clarity and succinctness will be rewarded.

Question 1: ____ (out of 10)
Question 2: ____ (out of 20)
Question 3: ____ (out of 10)
Question 4: ____ (out of 10)
Question 5: ____ (out of 10)
Question 6: ____ (out of 10)
Question 7: ____ (out of 10)
Question 9: ____ (out of 10)
Question 9: ____ (out of 10)

Total: ____ (out of 100)

1. Give a *deterministic finite state automaton* (FA) that accepts all strings over $\{a, b\}$ that contain the sub-word ab at least twice. Thus $aabaaaabab, abbbbabb$ are in the language and $baabb, abbba$ are not.



20 pts

2. a) Does the following hold? For every regular language L, every subset of L is regular as well. If not, then give a simple counterexample.

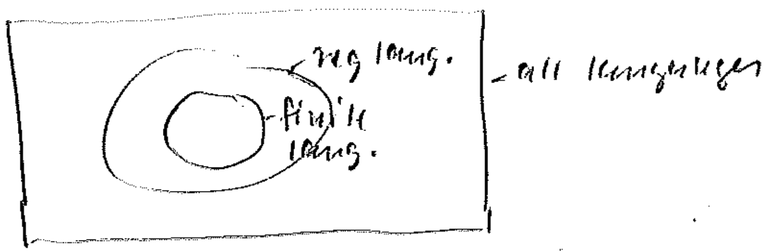
5 pts

No! $\underbrace{(a+b)^*}_{\text{reg}} \supseteq \underbrace{\{a^n b^n : n \geq 0\}}_{\text{non reg.}}$

b) Every non-regular language is infinite! True or false? Justify with a sentence.

5 pts

Yes! All finite languages are regular



c) The intersection of any two non-regular languages is non-regular. Is this true or false? If false, then give a counterexample.

5 pts

The intersection of two non-regular languages may be $\emptyset$, which is regular

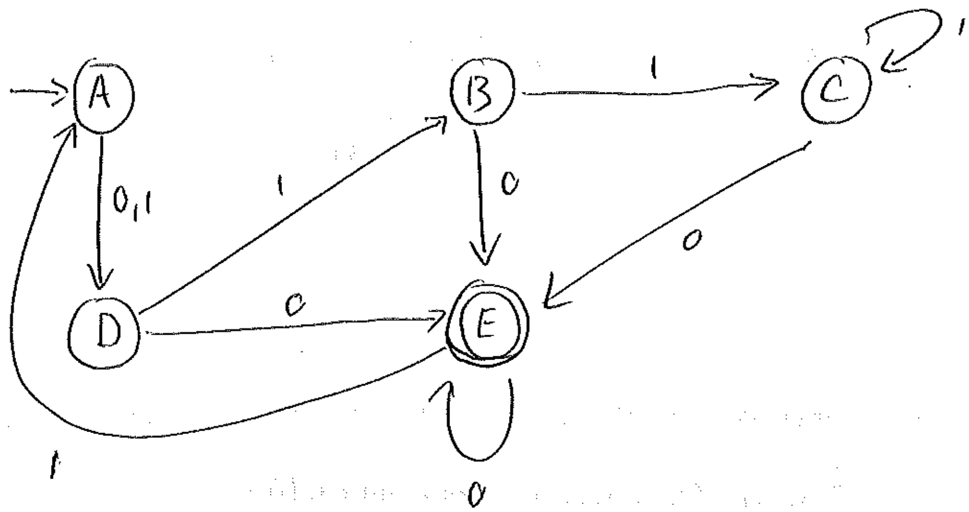
NO! $\underbrace{\{a^n b^n : n \geq 0\}}_{\text{non-reg}} \cap \underbrace{\{c^n d^n : n \geq 0\}}_{\text{non-reg}} = \underbrace{\emptyset}_{\text{reg}}$

d) Assume you are given two deterministic finite state automata, each with five states, that are known accept the same language. First you determine that the two machines are not isomorphic (i.e. identical except for renaming the states). What can you conclude from this already?

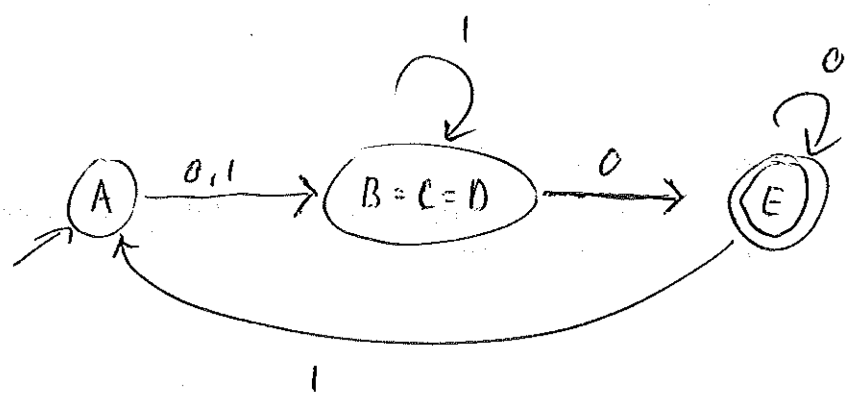
5 pts

The minimum state FA has < 5 states

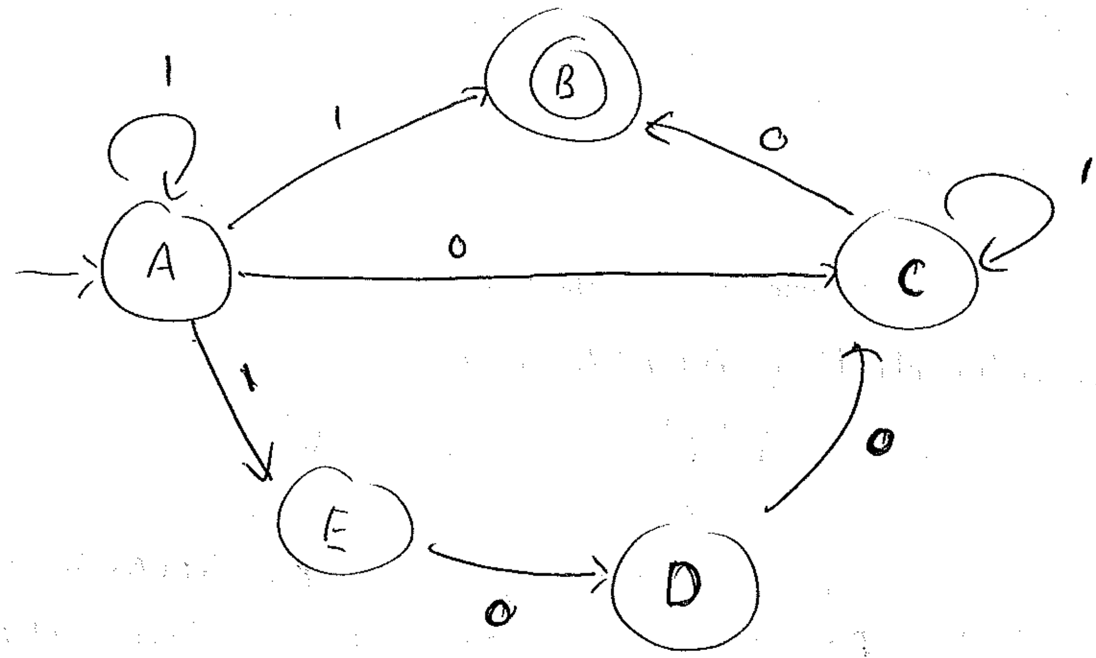
3. Minimize the following FA. Show your work (give the table). Show the resulting FA (if the number of states was reduced).

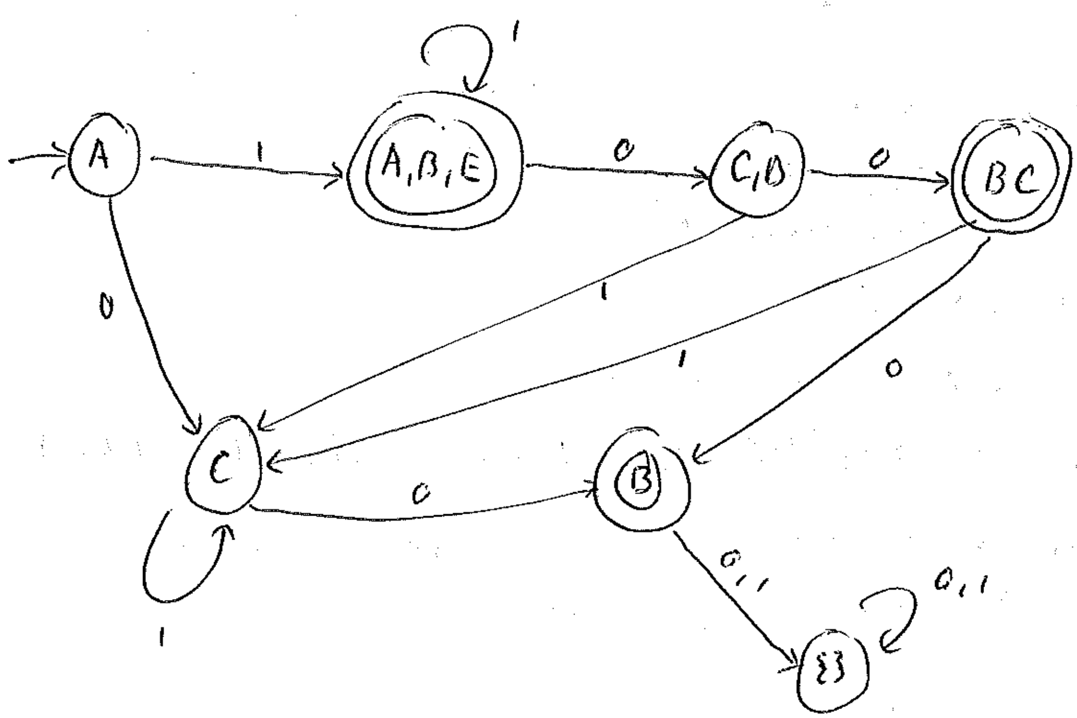


	A	B	C	D
B	2			
C	2			
D	2			
E	1	1	1	1



4. Use the "subset construction method" to convert the following NFA to an FA. Label the FA you produce with the corresponding subsets of the states of the original NFA.





5. Show that the language $L = \{0^i 1^j : i \geq j\}$ is non-regular by either using the following version of the Pumping Lemma or by exhibiting an infinite set of pairwise distinguishable words (you need to show that any pair in the set is distinguishable).

For every regular language L there is a constant N such that each word $x \in L$ of length at least N can be written as uvw such that the following holds:

$|uv| \leq N$,

v is not the empty word and

for all $i \geq 0$, $uv^iw \in L$.

Hint: Make sure that you are pumping in the right direction.

0^* pairwise distinguishable set

$\forall\ 0 \leq i < j$: $\quad 0^i \underbrace{1^j}_{z} \quad \wedge \quad 0^j \underbrace{1^j}_{z}$

$\notin L$ because $i < j$ $\qquad \in L$ because $j \geq j$

Thus $|\{0^i 1^j, 0^j 1^j\} \cap L| = 1$ and 1^j distinguishes 0^i from 0^j

Assume L reg.

Then P.L. applies

Let N be constant of PL

Choose $x = 0^N 1^N$

Since $x \in L \wedge |x| \geq N$

x can be written as uvw s.t. i) – iii) holds

Since x begins with 0^N and $|uv| \leq N$,

$uv \in 0^*$. By ii) $v \in 0^+$.

By iii) $uv^0w = uw = 0^{N-|v|} 1^N \in L$.

But since $|v| \geq 1$, $N - |v| < N$ and

therefore $uv^0w \notin L$.

Contradiction to assumption that L reg.

6. Use the closure properties of regular languages to show that the following languages are not regular:

a) The set of non-palindromes, i.e. the language $\underbrace{\{a,b\}^* - \overbrace{\{w \in \{a,b\}^* : w = w^R\}}^{PAL}}_{\overline{PAL}}$

Assume $\overline{PAL}$ reg.

Then since reg. lang. are closed under complementation, $\overline{\overline{PAL}} = PAL$ reg.

However PAL is known to be non-regular.

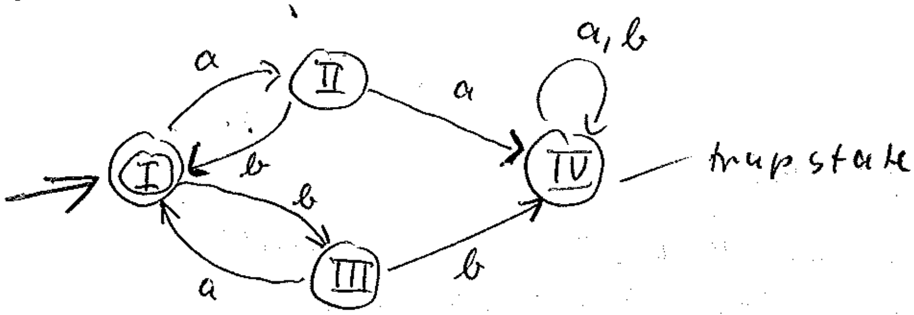
Contradiction!

Thus $\overline{PAL}$ must be non-reg.

b) Give six closure properties of regular languages.

- $\bullet$
- $\cup$
- $*$, $+$
- $\setminus$ set difference
- PREFIX, SUFFIX, SUBWORD
- REVERSE
- $-$ COMPLEMENT
- If L reg. then L^2 reg.

7. Show that there are four pairwise distinguishable word w.r.t. the language accepted by the following FA. What does this mean about the FA?



I	II	III	IV
λ,	a,	b,	aa

} picked one word per state

λ is distinguished from rest by λ

aa " " " a, b by b, a resp.

a " " " b by b.

Conclusion: At least 4 states are needed and above FA has minimum # of states

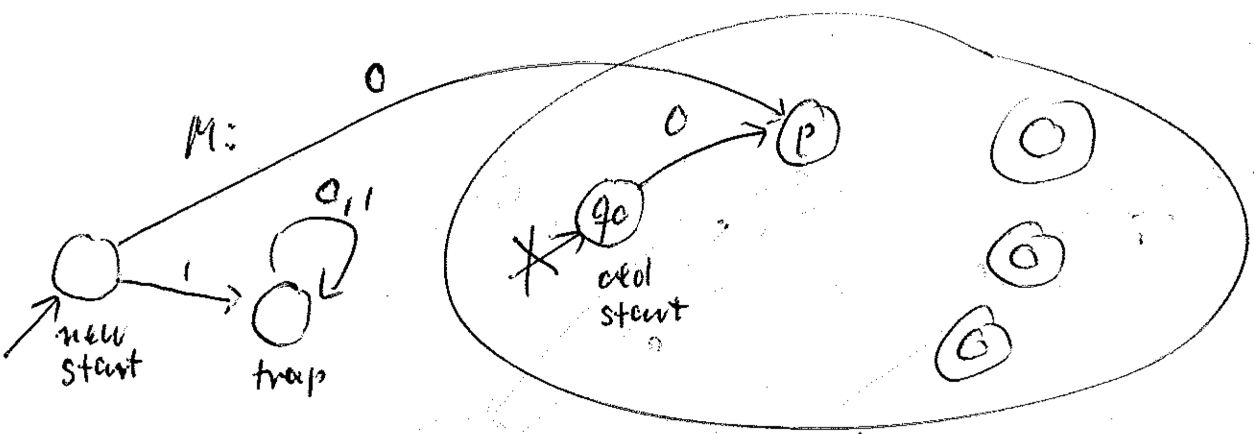
⊆

8. Show that for any regular language $L \cancel{\notin} \{0,1\}^*$, show that the following language is regular: $\tilde{L} = \{w \in L : w \text{ begins with } 0\}$.

1) $\tilde{L} = L \cap \underbrace{0(0+1)^*}_{\text{reg.}}$

$\tilde{L}$ reg. since reg. lang. closed under $\cap$

2) Modify FA M for recognizing L



p could be q0 !

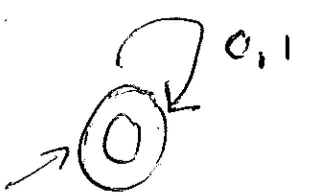
- undesignate old start state
- 0 succ. of new start is 0 succ p of old start state.
- 1 " " " " " trap state

3) A false solution:

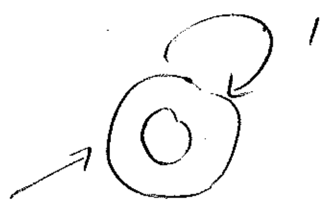
Let M be FA accepting L

Delete 0 transition out of start state

incorrect:

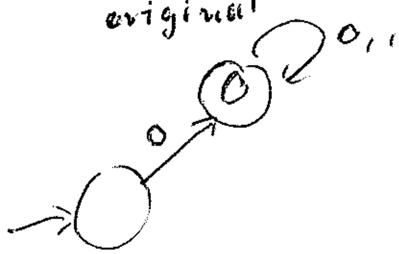


$L = (0+1)^*$
original

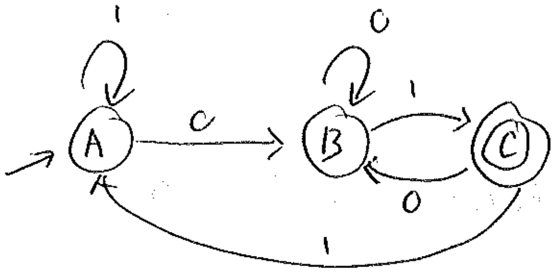


accepts 1^*
modified

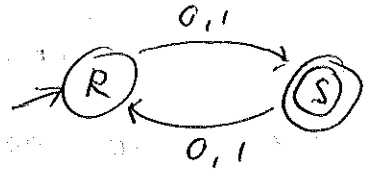
correct:



9. Using the "cross-product construction" method, build an FA for the intersection of the regular languages accepted by the following two FA's.



SECOND TO LAST BIT IS ZERO



ODD LENGTH WORDS