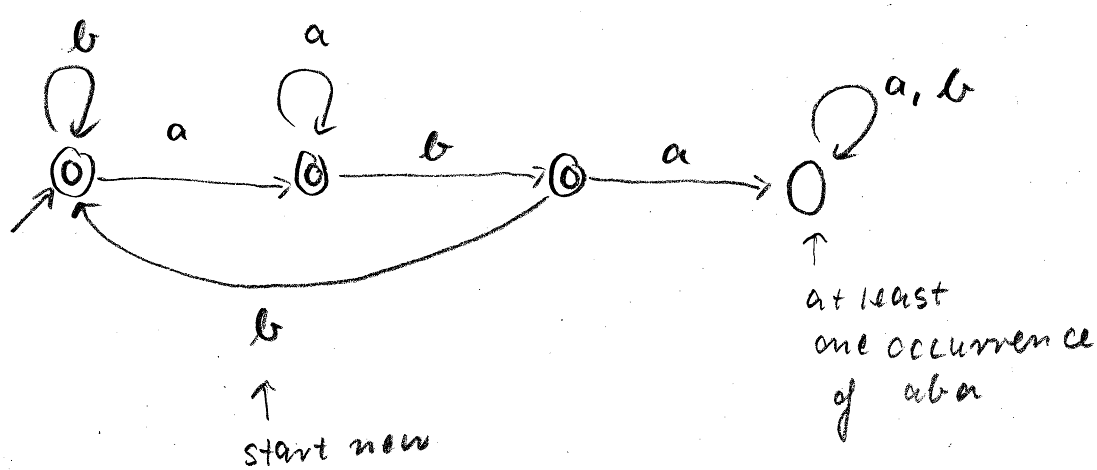
1. Give a deterministic finite state automaton (FA) that accepts all strings over $\{a, b\}$ that don't contain the substring aba.

15pts Hint: First construct an FA for all strings that contain aba.

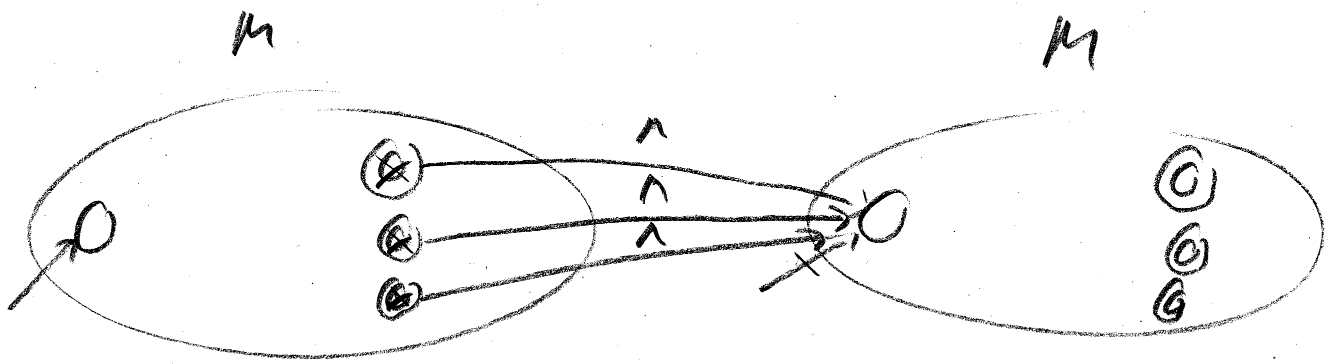


2)
15 pts

Show that for any regular language L, L^2 is regular as well, where $L^2 = \{vw : v \in L \text{ and } w \in L\}$.

1) Let r be reg. expr. for L
Then rr generatest L^2

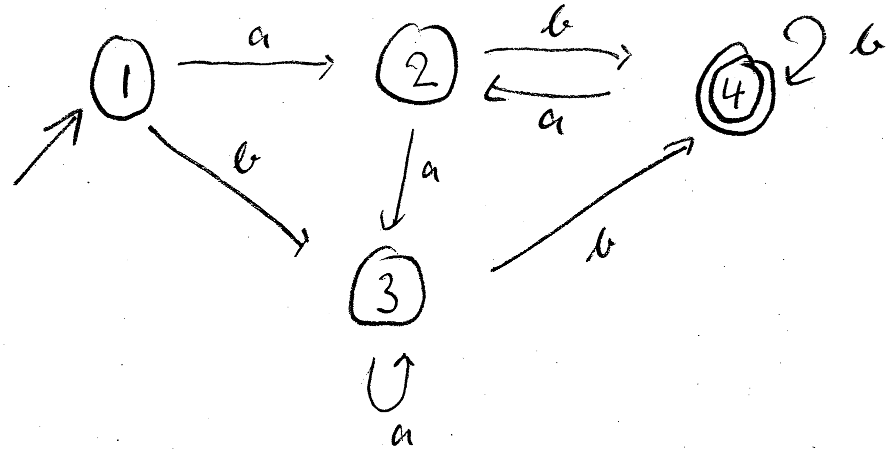
2) Given an NFA M for accepting L



- 2 copies of M
- λ transitions from final of first to start of second
- undesignate finals of first & start of second

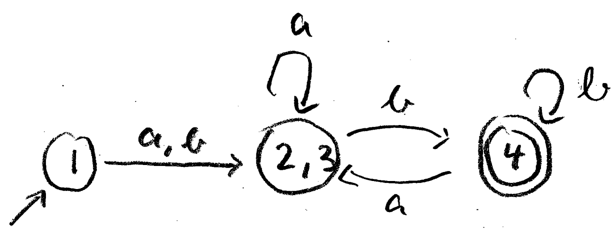
3) Minimize the following FA. Show your work (give the table). Show the resulting FA in case the number of states was reduced.

20 pts



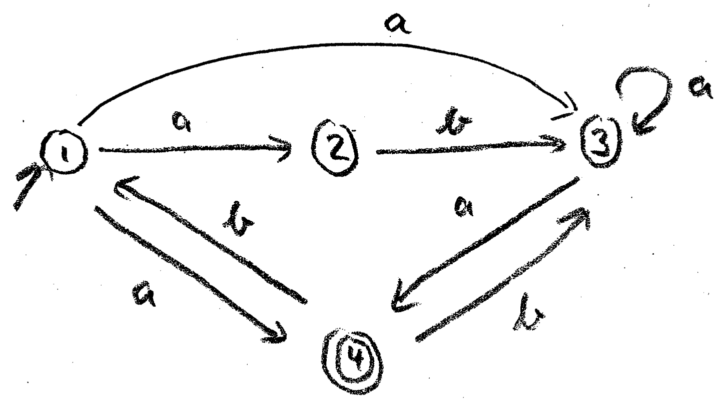
2	2		
3	2		
4	1	1	1
	1	2	3

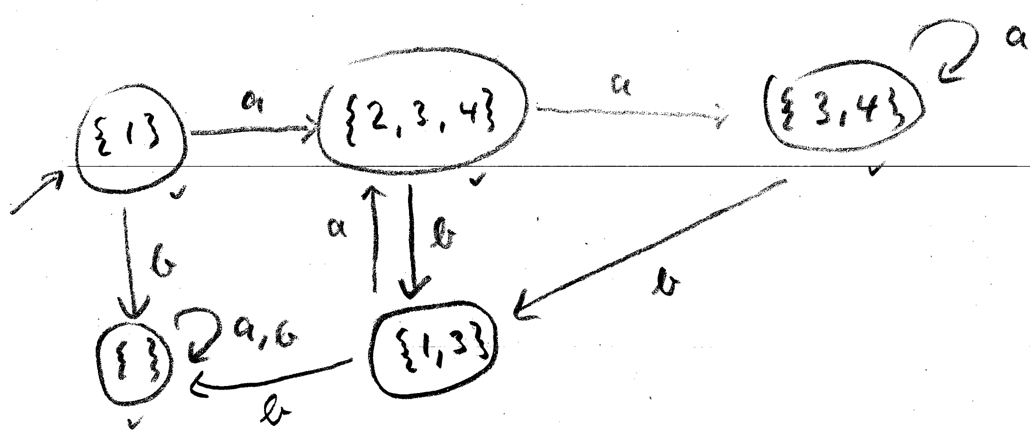
2 = 3



4) Use the "subset construction method" to convert the following NFA to an FA. Label the FA you produce with the subsets of states of original NFA.

20pts





5) 7. Give a pair of distinguishable words w.r.t. the language $L = 0(0+11)^*1$ and show that the pair is distinguishable.

15pts Hint: Choose a pair of short words

Λ and 01 are dist.

because $|\{\underset{\notin L}{\Lambda z}, \underset{\in L}{01z}\} \cap L| = 1$ for $z = \lambda$

6) Use the closure properties of regular languages to show that the following language are not regular:

15 pts

$$L = \{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ equals the number of } b\text{'s}\}$$

Hint: What simple non-regular language L' is L related to?

Assume L regular

Then $L \cap a^*b^*$ regular because

- reg. languages are closed under intersection
- a^*b^* is regular

But $L \cap a^*b^* = \{a^n b^n : n \geq 0\}$

which is not regular

This is a contradiction and therefore L is not regular

Extra Credit:

7) Show that the language $L = \{0^i 1^j 0^k : k \le i + j\}$ is non-regular using the following version of the Pumping Lemma.

15pts For every regular language L there is a constant N such that each word $x \in L$ of length at least N can be written as uvw such that the following holds:

i) $|uv| \le N$,

ii) v is not the empty word and

iii) for all $i \ge 0$, $uv^i w \in L$.

Hint: Try to pump up or down and check which way you arrive at a word that is not in L.

Assume L reg. Then PL applies. Let N be constant of PL.

Choose $x = 0^N 1 0^{N+1}$.

Since $x \in L$ and $|x| \geq N$

x can be rewritten as uvw s.t. i) - iii) hold

By i) $\wedge$ ii) $v = 0^q$ for $q > 0$

By iii) $uw = 0^{N-q} 1 0^{N+1}$

That is, for this word

$i = N - q$

$j = 1$

$k = N+1$

Since $k > i + j = N + 1 - q$,

$uw \notin L$ and we have a contradiction