$\boxed{8.21}$ CMPS 130, Fall 08, HW6 Krishna

15 pts

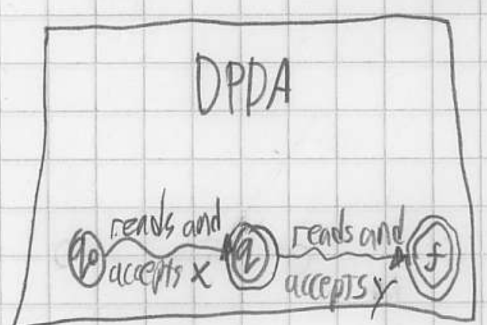
Let $x \in L$ and $xy \in L$. Since x is accepted by M, there exists a sequence of configurations

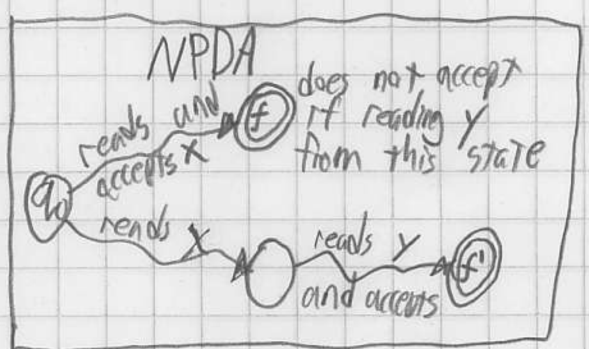
$$(p_0 = q_0, x_0 = x, \alpha_0 = Z_0) \vdash (p_1, x_1, \alpha_1) \vdash (p_2, x_2, \alpha_2) \vdash \cdots \vdash (p_k, x_k = \Lambda, \alpha_k)$$

where $p_k \in A$. Also, since xy is accepted by M, there is another configuration sequence

$$(r_0 = q_0, y_0 = xy, \beta_0 = Z_0) \vdash (r_1, y_1, \beta_1) \vdash \cdots \boxed{(r_n, y_n = y, \beta_n)} \vdash \cdots (r_m, y_m = \Lambda, \beta_m)$$

where $r_m \in A$. Note that the boxed configuration has read all of x in xy. In a DPDA, we would have that $p_k = r_n$ and $\alpha_k = \beta_n$ because it is deterministic. But this need not be the case for nondeterministic. For NPDA, that computation that accepts x may be disjoint. In a diagram this looks like



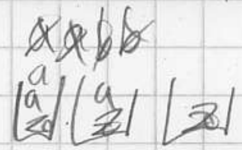


In other words, the machine may have to decide early on if it is looking at x or xy and going down the x path prevents it from considering the path that accepts xy.

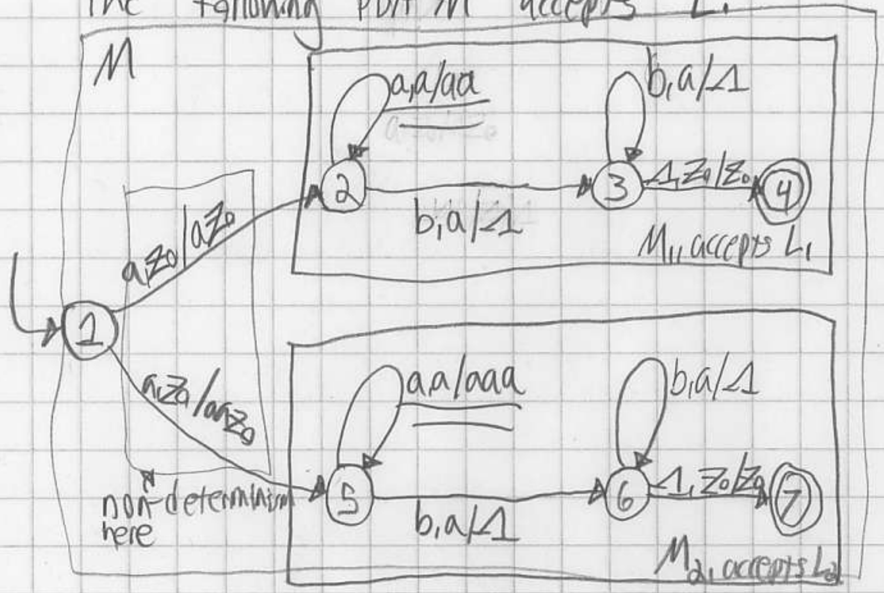
So to find a counterexample we should look for a NPDA that makes a choice early on as to which class of words it is reading and that choice cannot be "undone."

cont

$\boxed{8.21}$ con't



Consider $L_1 = \{a^n b^n \mid n\}$ and $L_2 = \{a^n b^{2n} \mid n\}$. Let $L = L_1 \cup L_2$.
The following PDA M accepts L:



push one a and match to every b

push two a's and match to every b.

Note how L how non-deterministically decides to try to recognize L_2 insted of L_1 by pushing two a's for every a on the input instead of trying to match L_1 and only pushing one a per a on input.

Now consider the machine M' constructed in the book running on $a^n b^n \# b^n$. $a^n b^n$ is in L and $a^n b^{2n}$ is also in L. Thus M' should accept $a^n b^n \# b^n$. The configuration sequence that accepts $a^n b^n$ stops in state 4 but there is no path from 4 to 7 that would allow the machine to accept $a^n b^{2n}$. Thus M' does not accept $a^n b^n \# b^n$ and thus does not accept $\{x\#y \mid x \in L, xy \in L\}$.