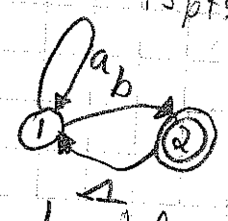
(c) The two FAs accept different languages. The left NFA-Λ accepts ab but the right NFA-Λ does not. 5 pts

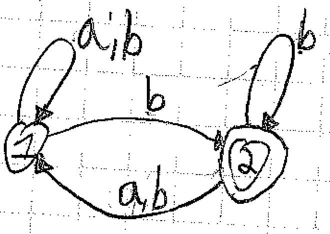
(d) The NFA-Λs accept the same language. To see this, we look at the <u>Λ-closure</u> of the left NFA-Λ. First we construct the table for δ^*: 15 pts



The left NFA-Λ

q	$\delta^*(q,\Lambda^*)$	$\delta^*(q,\Lambda^* a)$	$\delta^*(q,\Lambda^* b)$	$\delta^*(q,\Lambda^* a \Lambda^*)$	$\delta^*(q,\Lambda^* b \Lambda^*)$
1	{1}	{1}	{2}	{1}	{1,2}
2	{1,2}	{1}	{2}	{1}	{1,2}

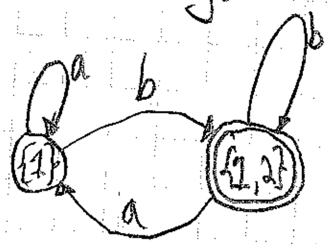
This produces the following NFA:



We can convert this into an FA using <u>subset construction</u>:

q	$\delta^*(q,a)$	$\delta^*(q,b)$
1	{1}	{1,2}
2	{1}	{1,2}

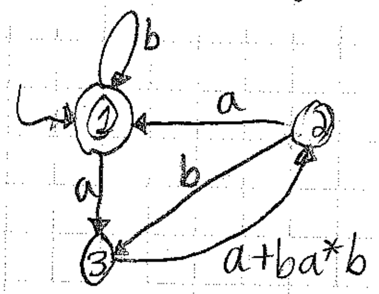
Thus we get



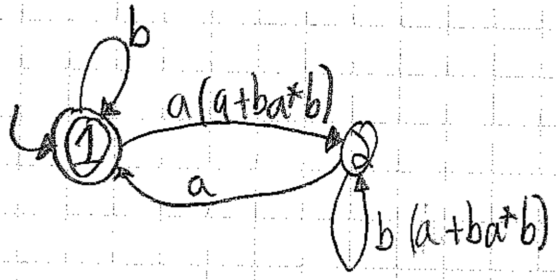
This the same FA as the one on the right, thus they accept the same language.

4.38

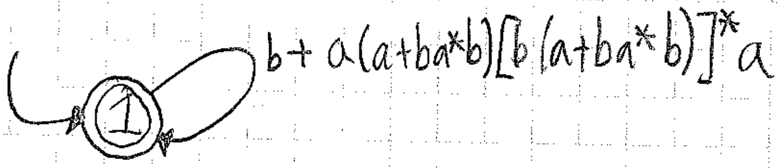
(a) First we eliminate state 4: 15 pts



Then eliminate state 3:



Now eliminate state 2:

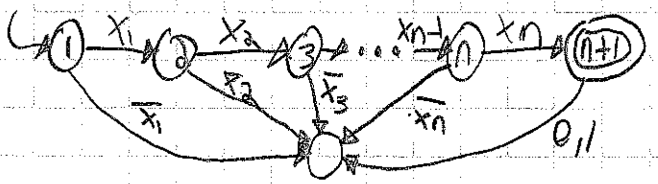


Thus the FA accepts

$$\{b+a(a+ba^*b)[b(a+ba^*b)]^*a\}^*.$$

$\boxed{5.2}$

Let $x = x_1 x_2 \cdots x_n$, where $x_i \in \{0,1\}$.
An FA that accepts this language is



20 pts

where

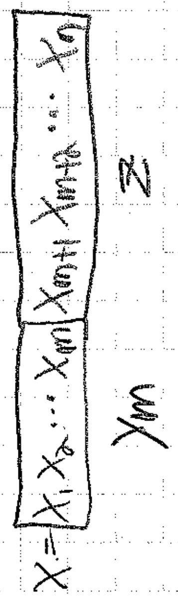
$$\bar{x}_i = \begin{cases} 1 & \text{if } x_i = 0 \\ 0 & \text{if } x_i = 1 \end{cases}$$

Looking at this FA suggests that each prefix

$$y_m = x_1 x_2 \cdots x_m \quad \text{where } m \le n$$

is distinguishable from every other prefix y_p. $(p \neq m)$.
To see this let $z = x_{m+1} \cdots x_n$, then

$$y_m z = x \in L \quad \text{but} \quad y_p z \neq x$$



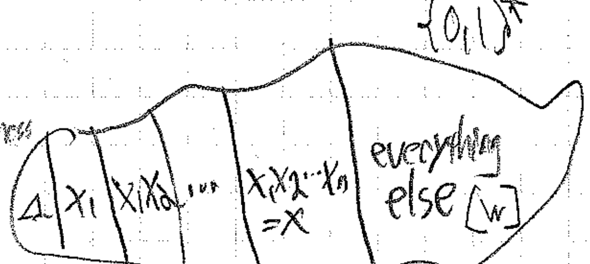
so $y_p z \notin L$. Thus $[y_m]$ are different equivalence classes for $m \le n$. Now consider any word that is not a prefix of x. These are indistinguishable since nothing can be concatenated to them to get x. Let w be any word that is not a prefix of x. Then $[w]$ is distinguishable from each $[y_m] = \{y_m\}$ because $z = x_{m+1} x_{m+2} \cdots x_n$ distinguishes any element of $[y_m]$ from $[w]$.

the only word in the language

We can let $w = \bar{x}_1$ unless $x = \Lambda$ then $w = 1$ will work fine

Thus the equivalence classes are

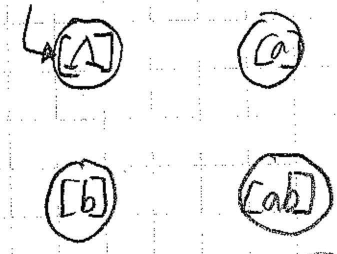
$$\underbrace{[\Lambda]}_{1 \text{ of these}},\ \underbrace{[x_1], [x_1x_2], \cdots, [x_1x_2\cdots x_n]}_{n \text{ of these}} \quad \text{and} \quad \underbrace{[w]}_{1 \text{ of these}}$$



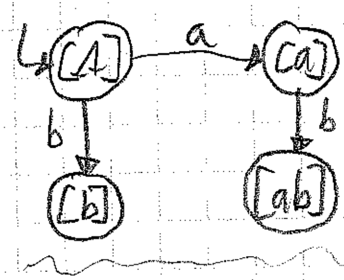
Thus there are $n+2$ equivalence classes. ■
What does this say about the FA shown above?

$\boxed{5.7}$ 15 pts

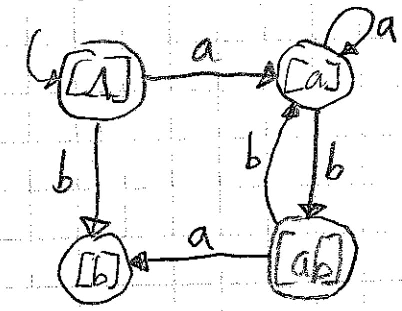
Because $[\Lambda]$, $[a]$, $[ab]$, and $[b]$ are all the equivalence classes, we know that the min. FA accepting L has 4 states. We lable the states with their class. $[\Lambda]$ is the initial state:



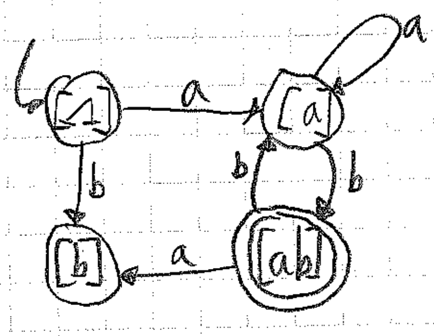
Because $\delta([x], c) = [xc]$ for all strings x and characters c, we can add the edges shown on the right:



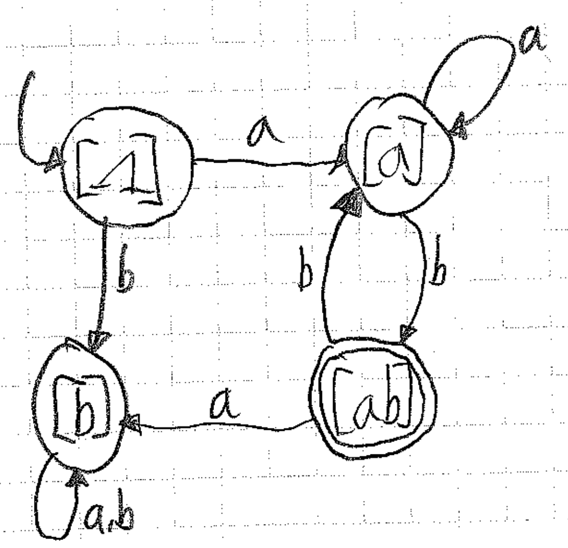
We are told $[a] = [aa] = [abb]$, thus $\delta([a], a) = [aa] = [a]$ and $\delta([ab], b) = [abb] = [a]$. Also $\delta([ab], a) = [aba] = [b]$. This gives:



$ab \in L$, means that $[ab]$ is an accepting state. Since $\Lambda \notin L$, $a \notin L$ and $b \notin L$, none of the other states are accepting.



Since b is not a prefix of any word in L, there can be <u>no</u> path from $[b]$ to $[ab]$. Thus no path from $[b]$ to $[\Lambda]$, $[a]$, or $[ab]$. Thus, all transitions from $[b]$ must to go a dead state. Since $[b]$ is already a dead state, we can add loops back to $[b]$ ~~that~~ read a and b. This gives the final FA:



$\boxed{5.16}$

ⓓ We build the table (as per Example 5.6):

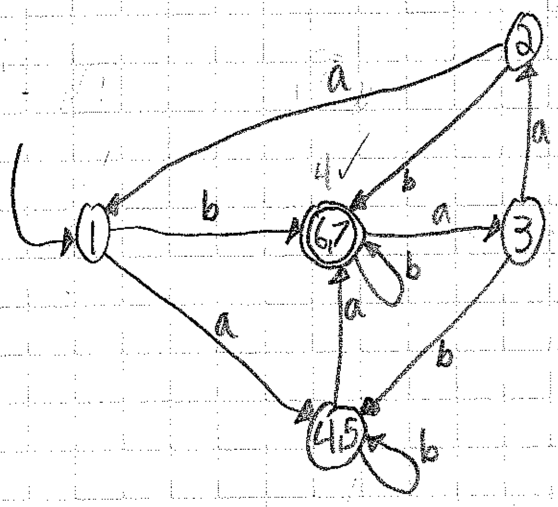
2	3^a					
3	2^b	2^b				
4	$2^{\substack{a\\b}}$	$2^{\substack{a\\b}}$	2^a			
5	$2^{\substack{a\\b}}$	$2^{\substack{a\\b}}$	2^a			
6	1	1	1	1	1	
7	1	1	1	1	1	
	1	2	3	4	5	6

$\boxed{2^{\substack{a\\b}}}$ } — symbols that moved the pair to prev. marked pair

└→ stage when mark

15 pts

Thus state 4 and 5 are in the same equivalence, as are 6 and 7. Thus we can colapse these states:



$\boxed{5.20}$

6

15 pts

(a) Consider 0^n and 0^m where $n \neq m$. Now let $z = 10^{2n}$. Thus $0^n z = 0^n 1 0^{2n} \in L$ but $0^m 1 0^{2n} \notin L$ since $n \neq m$. Thus z distinguishes 0^n and 0^m.