

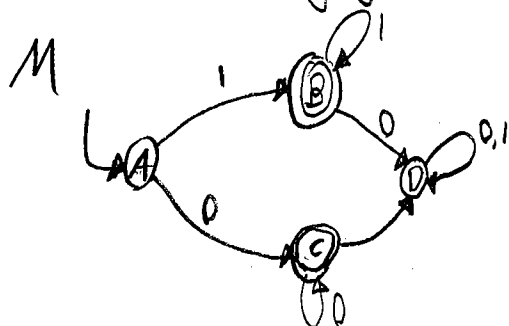


3.24

15 pts

2

Consider the language  $L$  over  $\{0,1\}$  accepted by



also  $11^* + 00^*$

Now consider the strings

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \end{aligned}$$

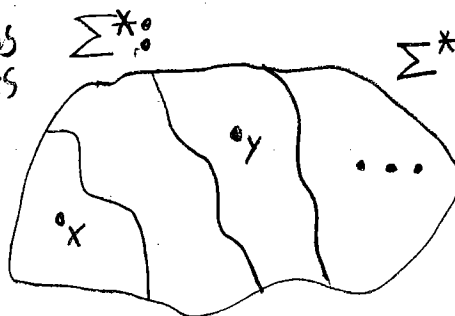
These are distinguishable (0 or 1 distinguishes them). Thus by Lemma 3.1

$$\delta^*(A, x_1) \neq \delta^*(A, x_2)$$

i.e. the machine must be in different states after read  $x_1$  and  $x_2$ . Since  $x_1 \in L$  and  $x_2 \notin L$ ,  $\delta^*(A, x_1)$  and  $\delta^*(A, x_2)$  must be accepting states. Thus any FA that recognizes  $L$  must have at least two accepting states.

In general: if  $L$  contains two distinguishable words (distinguishable with respect to  $L$ ), then any FA recognizing  $L$  must have at least two states.

The equivalence relation  $I_L$  partitions  $\Sigma^*$  if any two strings from different classes are not in  $L$ , then any FA that accepts  $L$  must have two or more accept states.



3.25

Let  $L$  be the language accepted by the FA.

3

Looking at the FA in Fig. 3.11d, consider the shortest string that will get you into each of the FA's four states:

$$\begin{array}{l} x_1 = \Delta \\ x_2 = 0 \\ x_3 = 01 \\ x_4 = 00 \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{land in accept state} \\ \\ \text{land in reject state} \end{array}$$

Of these  $x_1$  and  $x_2$  are accepted by the FA and  $x_3$  and  $x_4$  are not accepted.

Thus  $x_1$  or  $x_2$  can be distinguished from  $x_3$  or  $x_4$  by  $\Delta$ .  $x_1\Delta, x_2\Delta \in L$  but  $x_3\Delta, x_4\Delta \notin L$ .

Within these two classes,  $0$  distinguishes  $x_1$  from  $x_2$ :

$$\begin{array}{l} x_10 = 0 \in L \\ x_20 = 00 \notin L \end{array}$$

and  $0$  also distinguishes  $x_3$  and  $x_4$

$$\begin{array}{l} x_30 = 010 \in L \\ x_40 = 000 \notin L \end{array}$$

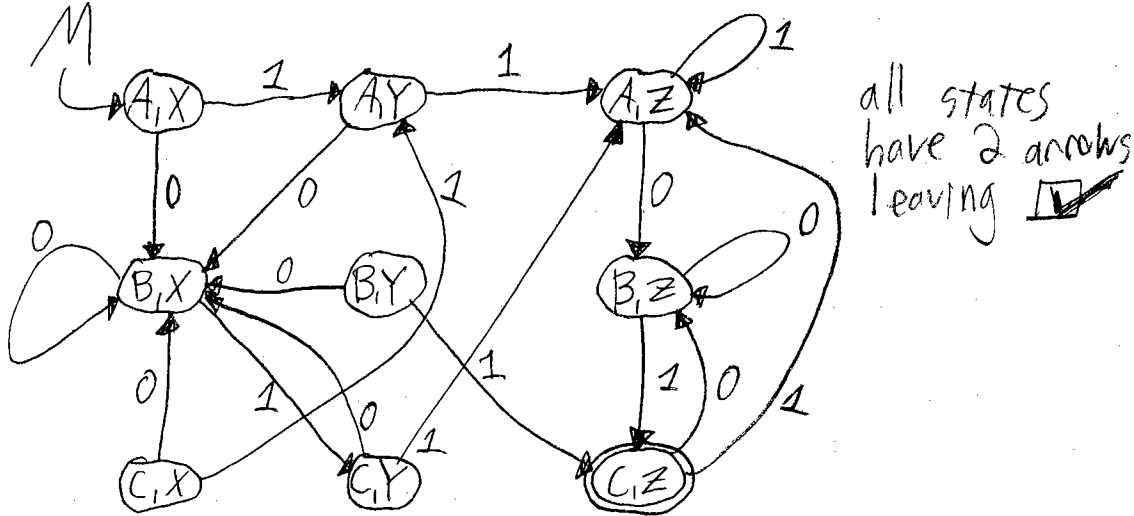
Thus, by Thm 3.2, any FA that accepts  $L$  must have at least four states.

3.33

20 pts

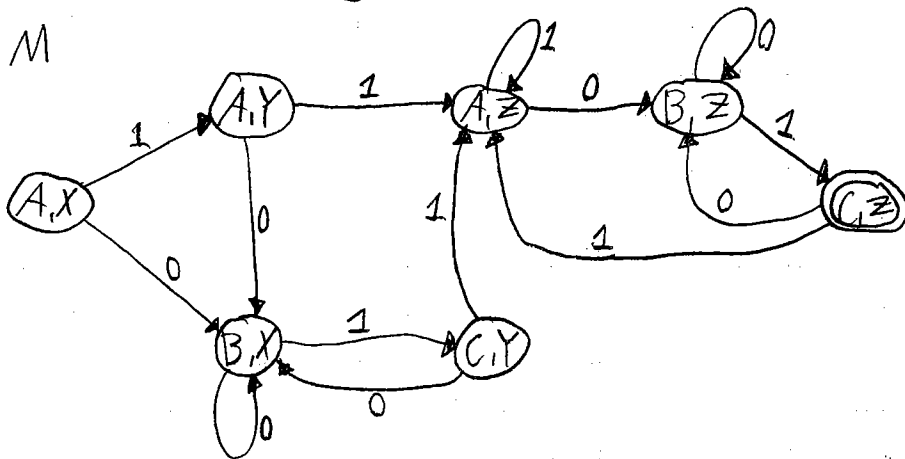
4

(b) Using the cross-product construction, we will produce an FA that has states corresponding to pairs of states from the original FAs:



Since we want to recognize  $L_1 \cap L_2$  the final states of  $M$  correspond to the states were  $M_1$  and  $M_2$  accept. This is state (C,Z).

Note that there are no arrows into states (C,X) and (B,Y). Thus these states can be removed. This gives:



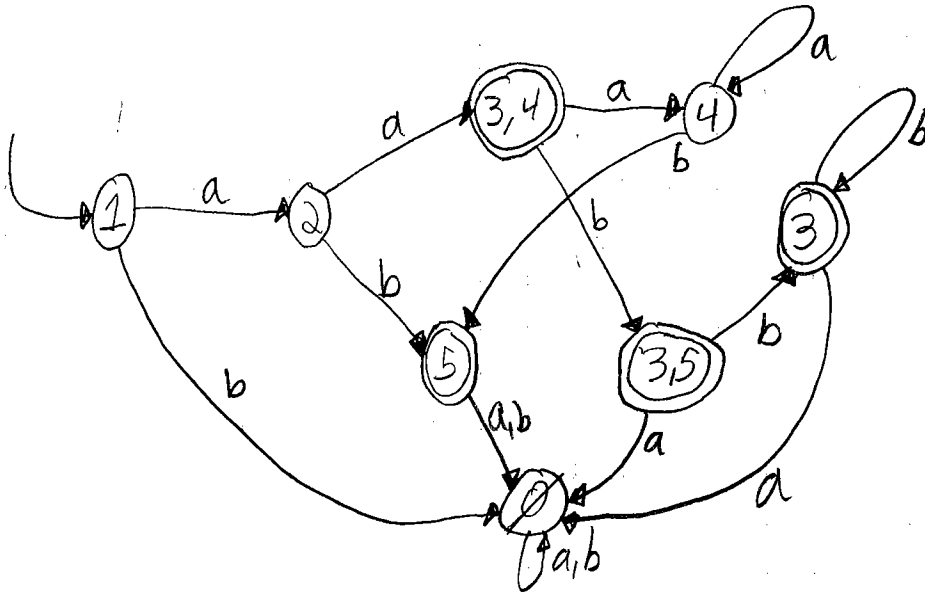
cross prod can

4.10f

15 pts

5

Using the subset construction, we will produce an FA that has states that correspond to subsets of states of the original NFA.

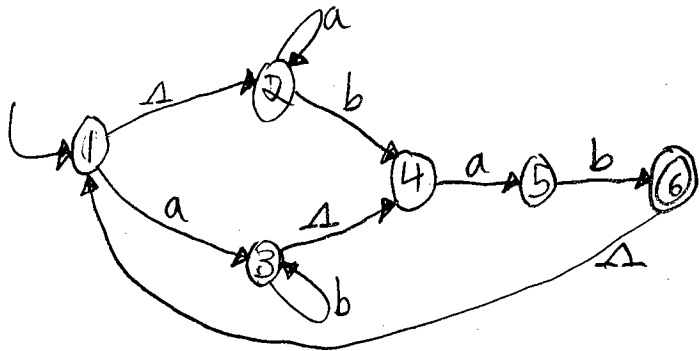


# Problem 6

20pts

6

The  $\Lambda$ -NFA is



Why are states always labeled starting with 1 and not 0?

The table is

$q$	$\delta^*(q, \Lambda^*)$	$\delta^*(q, \Lambda^*a)$	$\delta^*(q, \Lambda^*b)$	$\delta^*(q, \Lambda^*a\Lambda^*)$	$\delta^*(q, \Lambda^*b\Lambda^*)$
1	1, 2	2, 3	4	2, 3, 4	4
2	2	2	4	2	4
3	3, 4	5	3	5	3, 4
4	4	5	$\emptyset$	5	$\emptyset$
5	5	$\emptyset$	6	$\emptyset$	1, 2, 6
6	1, 2, 6	2, 3	4	2, 3, 4	4

new edges given here

Now we removed the transitions and add the ones given by the last two columns:

