

CMPS 102 Homework 6, Spring '04

4 problems, 25 points, due Thursday June 3rd

1. (4 pts) Exercise 16.1-2 in the text. Show that selecting the last compatible activity to start (instead of the first compatible activity to finish) is a greedy algorithm that finds a maximum set of compatible activities.
2. (5 pts) Coin changing. Recall that the coin changing problem is:

Given a set of n denominations $d_1 = 1 < d_2 < d_3 < \dots < d_n$ and a value C to give in change, find a way to give value C in change using as few coins as possible. Assume that there are as many coins as needed of each denomination.

One greedy algorithm takes as many high-valued coins as possible, and then moves to the next lower denomination. This doesn't always give an optimal solution, as the counterexample $C = 16, d_1 = 1, d_2 = 8, d_3 = 10$ shows. Find another counterexample showing that this greedy algorithm is not optimal even when each $d_i \leq d_{i+1}/2$.

3. (8 pts) Home Video CD. Assume you have n home video clips V_1, V_2, \dots, V_n where each clip V_i is m_i minutes long. Assume that the clips are numbered shortest to longest, so $m_1 < m_2 < m_3 < \dots < m_n$. You would like to store as many of the clips as possible on a single CD that can hold C minutes of video ($C \ll \sum_i m_i$, so they don't all fit). Assume that each video clips can only be stored in their entirety.

One greedy algorithm takes clips "shortest first" with the following pseudo-code (recall that the clips are sorted "shortest first").

1. `timeLeft = C;`
2. `for i = 1 to n do`
3. `if $m_i \leq \text{timeLeft}$ then`
4. `take clip V_i`
5. `timeLeft = timeLeft - m_i`

Either prove that the greedy algorithm that picks clips shortest first always maximizes the number of clips stored on the CD or find a counterexample.

4. (8 pts) Consider a modification to the above problem where the goal is to fill up as many minutes of the CD as possible (copying each clip at most once). Either prove that the following greedy algorithm taking the longest clip that will still fit on the CD always takes as many minutes of video as possible or find a counterexample.

1. `timeLeft = C;`
2. `for i = n downto 1 do`
3. `if $m_i \leq \text{timeLeft}$ then`
4. `take clip V_i`
5. `timeLeft = timeLeft - m_i`