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Motion Analysis and Optical Flow

CMPE 290V: Advanced Topics in Visual Computing

Hai Tao

Optical flow equation

- Tow assumptions
  - Assume the apparent brightness of moving objects remains constant between frames
  - Assume the image brightness is continuous and differentiable
- The optical flow equation is

$$I_x u + I_y v + I_t = 0$$

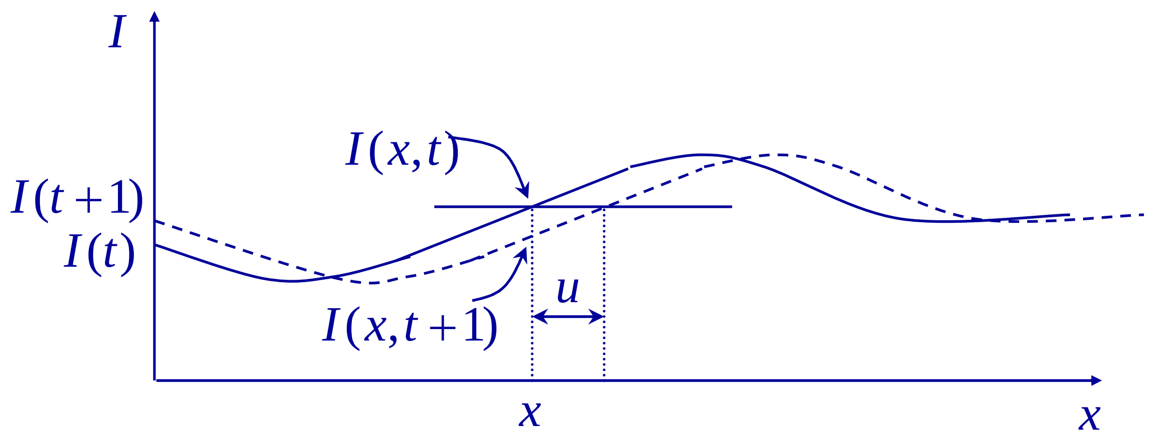
where I_t is the image difference between the two images, and I_x and I_y are image derivatives.

Geometric interpretation

- At time t, the local image brightness surface around (x,y) is modeled as a 3D plane with slope I_x in the x direction and I_y in the y direction Suppose from time t to t+1, the brightness surface plane moves (u,v), then the brightness change at (x,y) I_t is $-(I_x u+I_y v)$, therefore

$$I_x u+I_y v+I_t=0$$

- A 1D illustration



$$I_x u=-(I(t+1)-I(t)) \quad \text{or} \quad I_x u+I_t=0$$

Estimating optical flow – Constant flow field

- Assumption – for each pixel, its neighboring pixels in a small (NxN) window has the same motion
- As the result

$$\begin{bmatrix} I_{x,1} & I_{y,1} \\ \vdots & \\ I_{x,N^2} & I_{y,N^2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -I_{t,1} \\ \vdots \\ -I_{t,N^2} \end{bmatrix}$$

or

$$A \begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{b}$$

If the rank of A is larger than 1, the linear system is not under-constrained. The solution is

$$\begin{bmatrix} u \\ v \end{bmatrix} = (A^T A)^{-1} A^T \mathbf{b}$$

Affine flow field

- Affine motion field (approximation to the homography)

$$u = a_1 x + a_2 y + a_3$$
$$v = a_4 x + a_5 y + a_6$$

For each pixel, the optical flow equation becomes

$$\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_6 \end{bmatrix} = -I_t$$

or

$$\begin{bmatrix} I_x x & I_x y & I_x & I_y x & I_y y & I_y \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_8 \end{bmatrix} = -I_t$$

Stack all the all the equation for all the pixels yields the linear system

$$G \begin{bmatrix} a_1 \\ \vdots \\ a_8 \end{bmatrix} = \mathbf{b}$$ the solution is $$\begin{bmatrix} a_1 \\ \vdots \\ a_8 \end{bmatrix} = (G^T G)^{-1} G^T \mathbf{b}$$

Quadratic flow field

- Quadratic motion field (approximation to the homography)

$$u = a_1 x^2 + a_2 xy + a_3 x + a_4 y + a_5$$
$$v = a_1 xy + a_2 y^2 + a_6 y + a_7 x + a_8$$

For each pixel, the optical flow equation becomes

$$\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} x^2 & xy & x & y & 1 & 0 & 0 & 0 \\ xy & y^2 & 0 & 0 & 0 & y & x & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_8 \end{bmatrix} = -I_t$$

or

$$\begin{bmatrix} I_x x^2 + I_y xy & I_x xy + I_y y^2 & I_x x & I_x y & I_x & I_y y & I_y x & I_y y \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_8 \end{bmatrix} = -I_t$$

Stack all the all the equation for all the pixels yields the linear system

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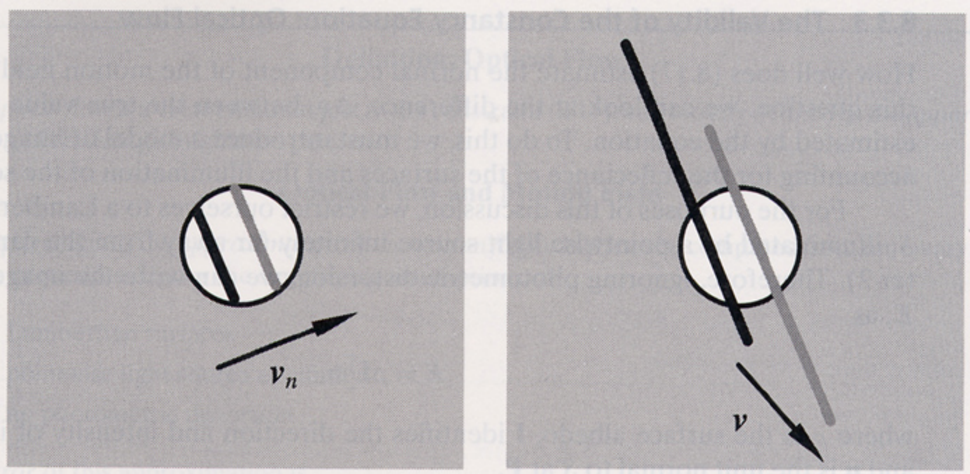
Problem (1-2) – Low texture and aperture effect

- In a textureless region, G will be 0, no or infinite solution
- G can also be rank 1. The motion that is orthogonal to the spatial image gradient direction can not be estimated in a local region. This is called aperture effect



(a) (b)

Figure 8.7 The aperture problem: the black and grey lines show two positions of the same image line in two consecutive frames. The image velocity perceived in (a) through the small aperture, $\mathbf{v}_n$, is only the component parallel to the image gradient of the true image velocity, $\mathbf{v}$, revealed in (b).

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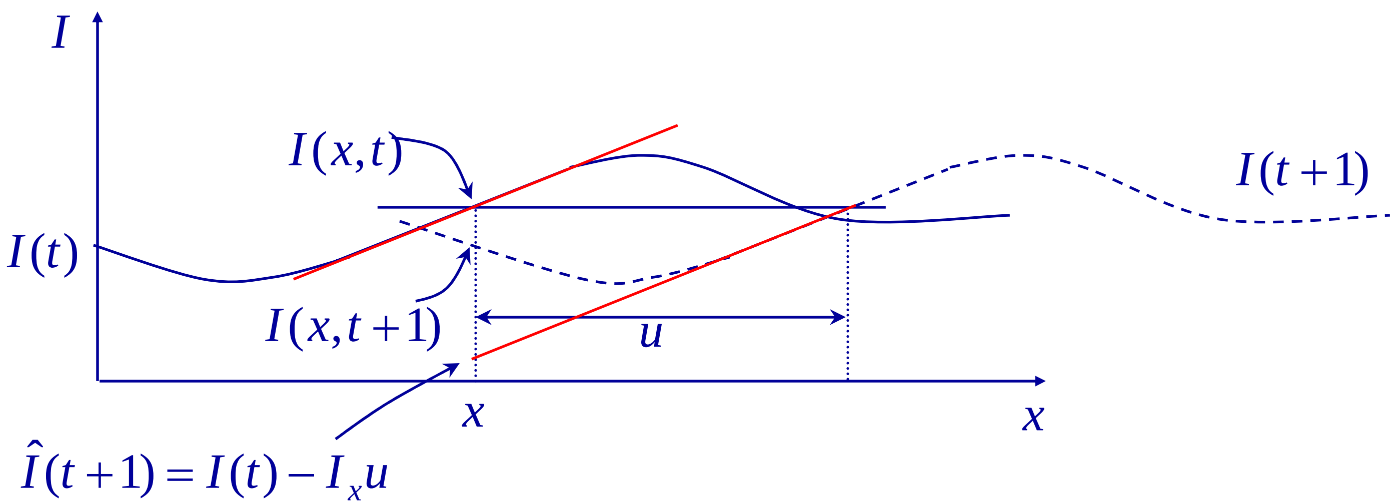
Problem (3) - Large motion

- From the previous illustration, we also observe that the optical flow equation is not a good approximation if the motion is too large



- Image motion estimation techniques based on the image brightness constancy equation is referred as optical flow techniques

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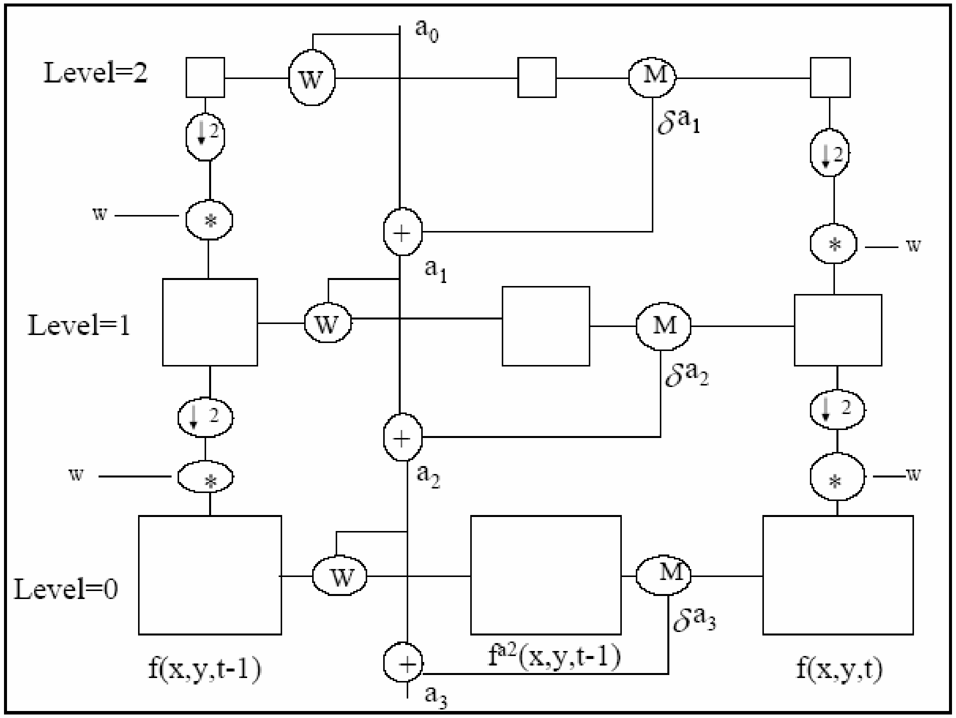
Solution – Coarse-to-fine motion estimation (Bergen et al, ECCV92)

- The main components of the hierarchical method are
  - Pyramid construction
  - Motion estimation
  - Image warping
  - Coarse-to-fine refinement
- Block diagram



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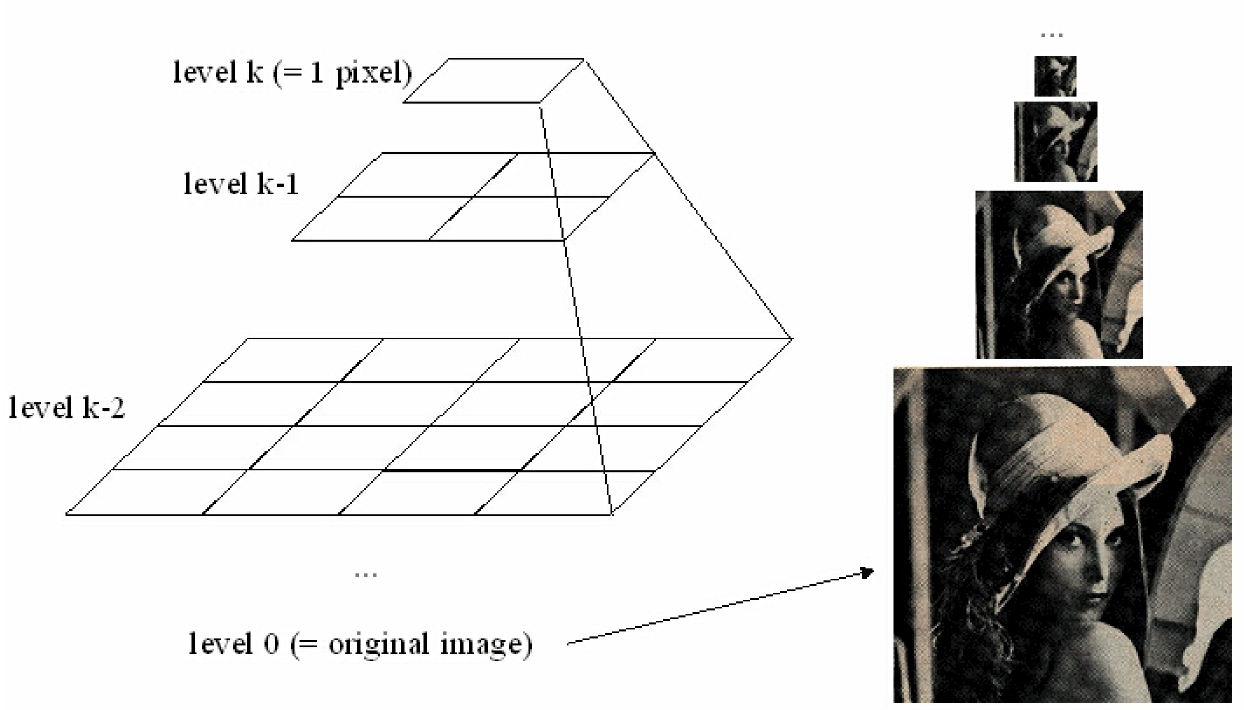
Pyramid construction (1)

- The Gaussian pyramid

$$P_{Gaussian}(I)_0 = I$$

$$P_{Gaussian}(I)_k = \downarrow G(P_{Gaussian}(I)_{k-1}) \quad \text{where } G() \text{ is a Gaussian filter,}$$

$\downarrow$ is a downsampling operator



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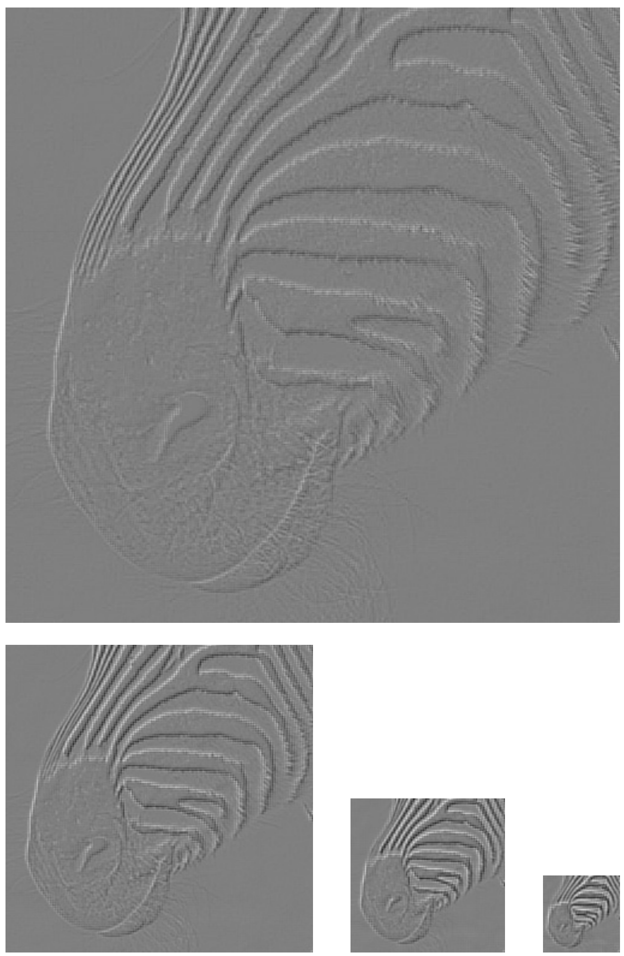
Pyramid construction (2)

- The Laplacian pyramid




$$P_{Laplacian}(I)_M = P_{Gaussian}(I)_M$$

$$P_{Laplacian}(I)_k = P_{Gaussian}(I)_k - \uparrow P_{Gaussian}(I)_{k+1}$$

$\uparrow$ is an upsampling operator

Gaussian vs. Laplacian pyramid

- The Laplacian pyramid is used when illuminate or appearance changes from image to image

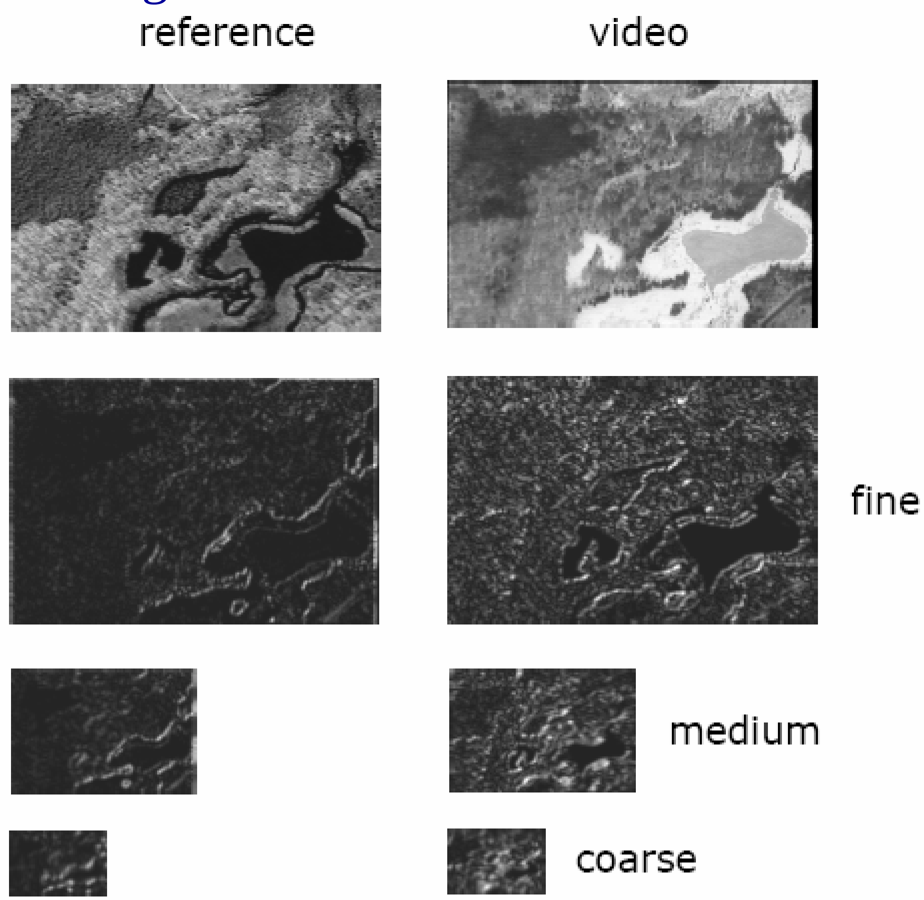


Image warping and image morphing (cont.)

- Image warping classification
  - Forward vs. backward
  - Triangular mesh or rectangular mesh or no mesh
  - Linear interpolation or spline interpolation

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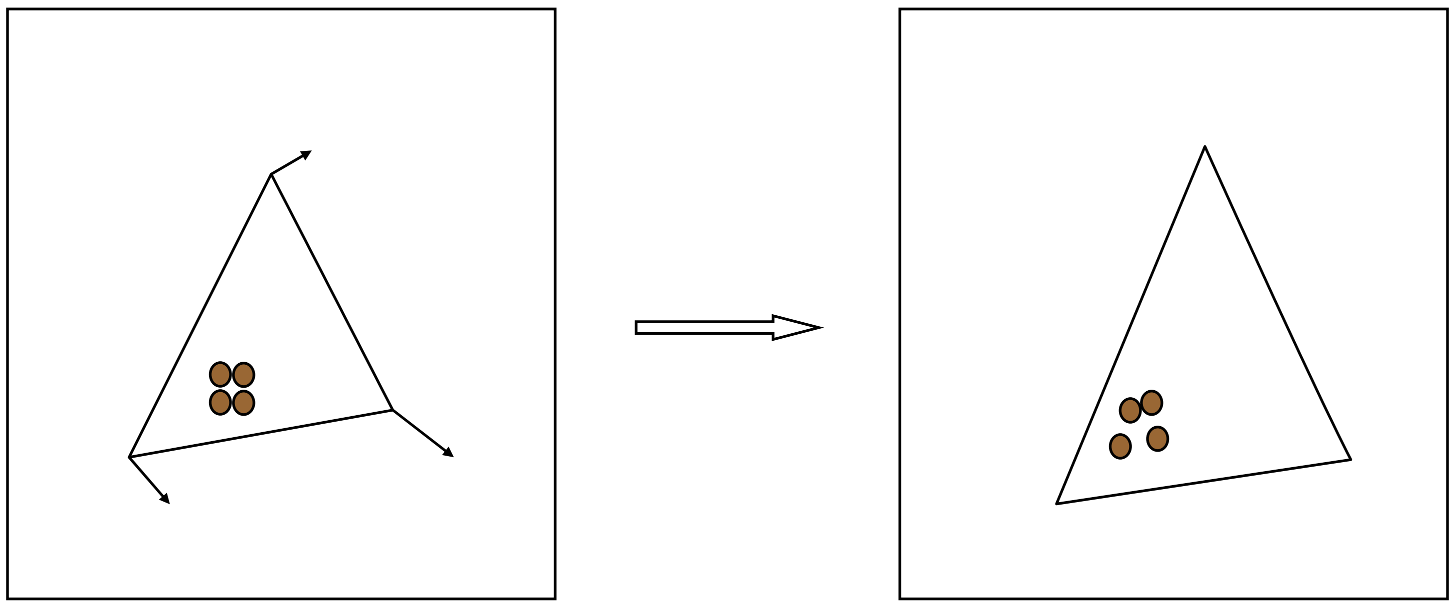
Forward warping and backward warping

- Forward warping - start from original image



original

target

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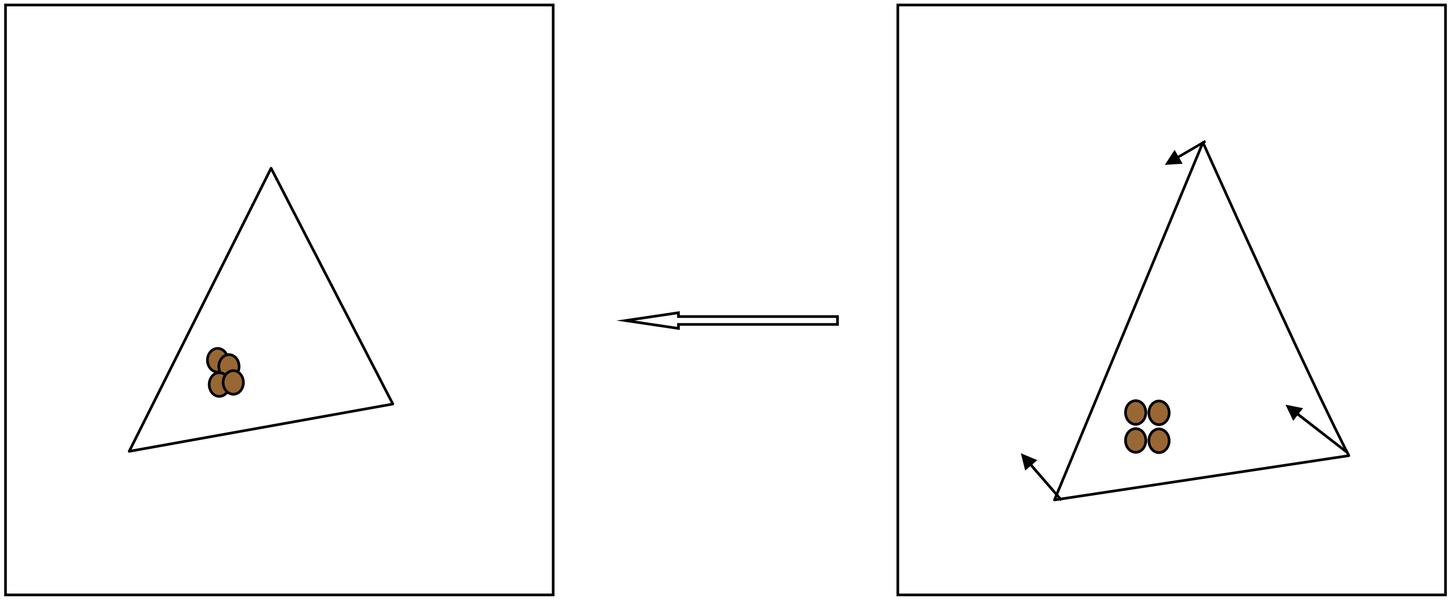
Forward warping and backward warping (cont.)

- Backward warping - start from target image



original

target

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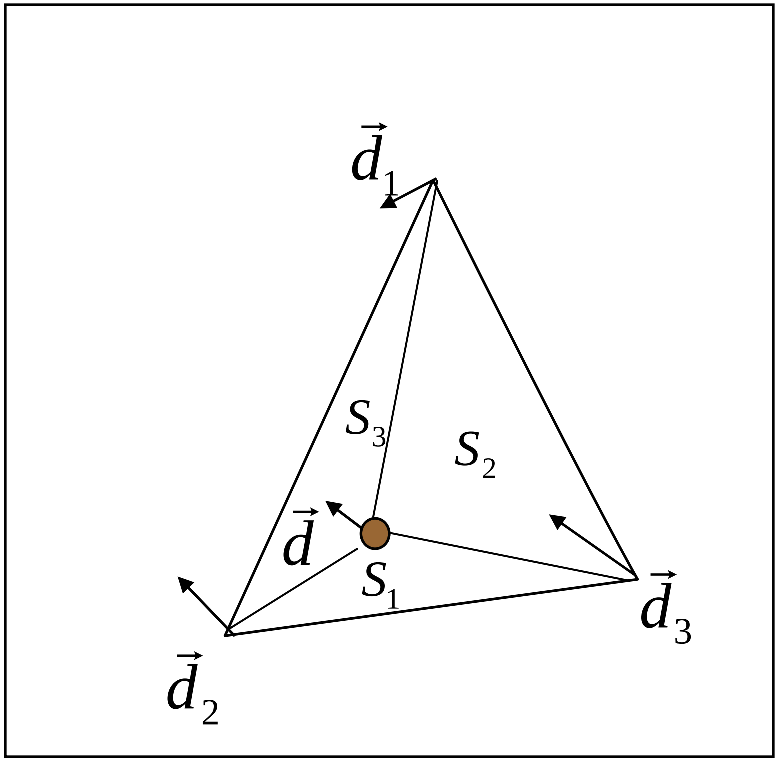
Displacement interpolation



$$\vec{d} = (S_1\vec{d}_1 + S_2\vec{d}_2 + S_3\vec{d}_3)/(S_1 + S_2 + S_3)$$

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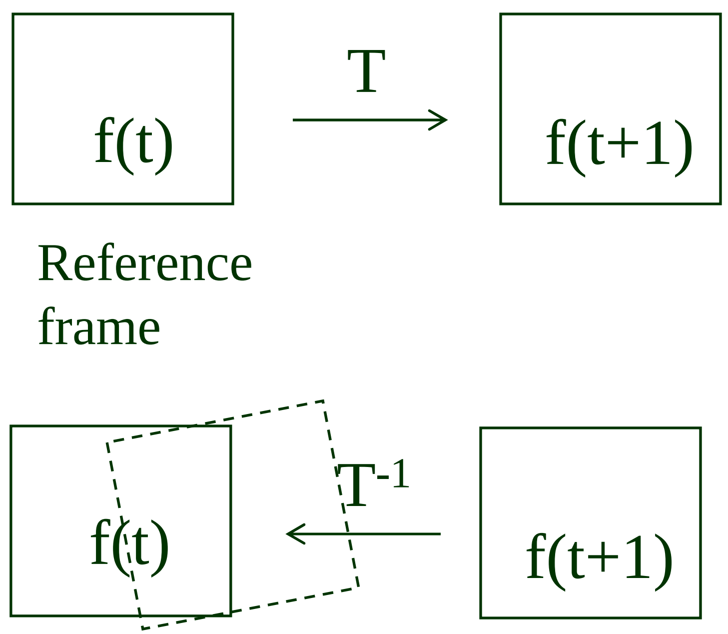
Image mosaicking and panoramic view

- Based on frame-to-frame transformation, multiple images can be "stitched" together
- Two-frame mosaicking



Removing stitching artifact

- Adelson, E.H., Anderson, C.H., Bergen, J.R., Burt, P.J., M., O.J.: Pyramid method in image processing. RCA Engineer **29(6)** (1984) 33–41.
- A. Levin, A. Zomet, S. Peleg and Y. Weiss. Seamless Image Stitching in the Gradient Domain. *Proc. of the European Conference on Computer Vision (ECCV), Prague, May 2004.*

Levin et al, ECCV'2004

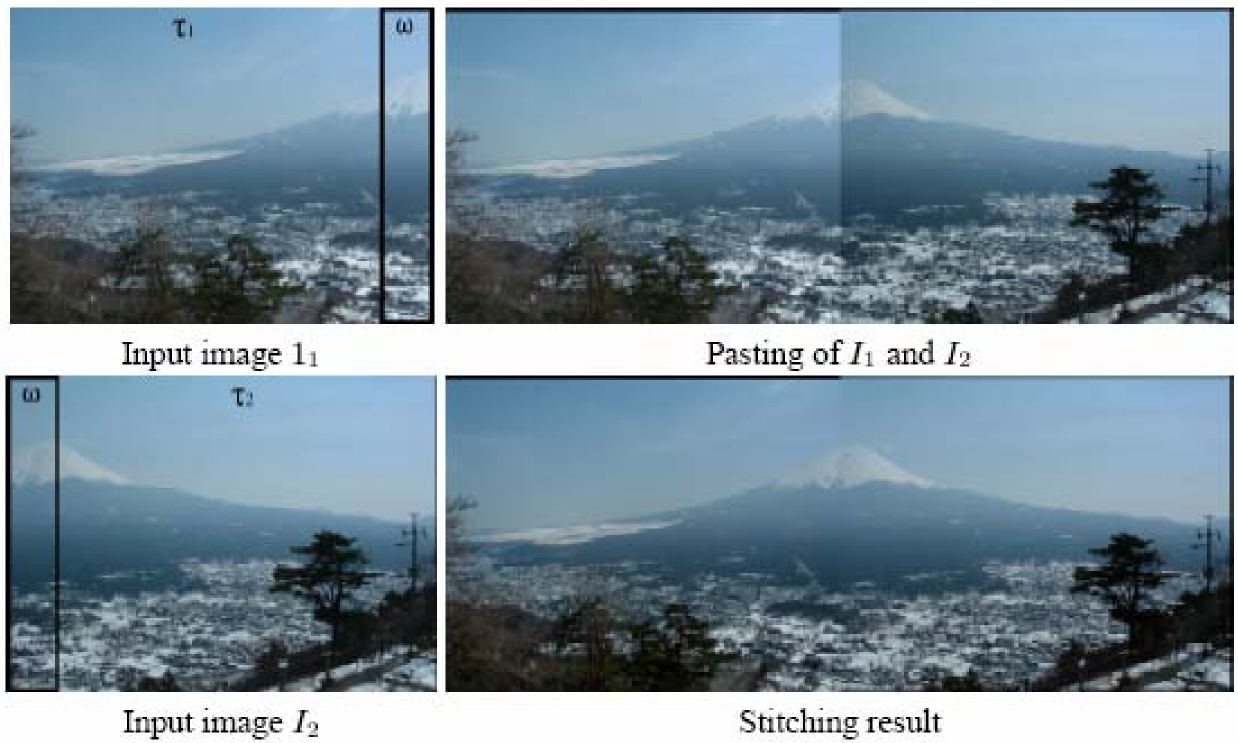


Fig. 1. Image stitching. On the left are the input images. ω is the overlap region. On top right is a simple pasting of the input images. On the bottom right is the result of the GIST1 algorithm.

Examples

From http://www.panavue.com/products/panavue_gallery.htm

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