Department of Computer Engineering
University of California at Santa Cruz

Image Noise and Filtering

CMPE 264: Image Analysis and Computer Vision

Hai Tao

Estimating acquisition noise

- Noise introduced by imaging system
- Simple model
  - Assumption: noise at each pixel is independent, characterized by its mean and standard deviation
  - Estimating the mean and the standard deviation σ for each pixel

$$\text{For each } i, j = 0, \ldots, N-1, \text{ let}$$

$$\bar{I}(i,j) = \frac{1}{N}\sum_{k=0}^{N-1} I_k(i,j)$$

$$\sigma(i,j) = \left(\frac{1}{N-1}\sum_{k=0}^{N-1}\left(I_k(i,j) - \bar{I}(i,j)\right)^2\right)^{1/2}$$

  - For most imaging system, $\sigma \approx 2.5$

Estimating acquisition noise

- Estimating auto-covariance
  - In reality, the noise in neighboring pixels is not independent
  - The correlation is described by auto-covariance
  - If we assume auto-covariance of noise is the same everywhere in the image, then

$$Let\ N_{i'} = N - i' - 1, N_{j'} = N - j' - 1,\ for\ each\ i', j' = 0, \ldots, N-1,\ compute$$

$$C_{II}(i', j') = \frac{1}{N^2} \sum_{i=0}^{N_{i'}} \sum_{j=0}^{N_{j'}} (I_k(i,j) - \bar{I}(i,j))(I_k(i+i', j+j') - \bar{I}(i+i', j+j'))$$

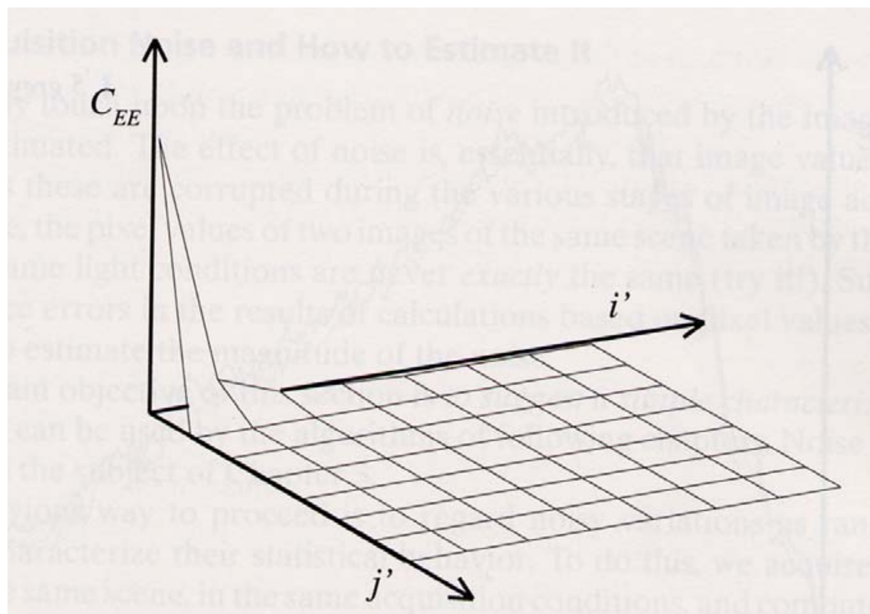
  - Example: How to compute $C_{II}(2,1)$ for a 10x10 image ?

$$N_{i'} = 7, N_{j'} = 8,\ for\ each\ i' = 2, j' = 1,\ compute$$

$$C_{II}(2,1) = \frac{1}{10^2} \sum_{i=0}^{7} \sum_{j=0}^{8} (I_k(i,j) - \bar{I}(i,j))(I_k(i+i', j+j') - \bar{I}(i+i', j+j'))$$

Estimating acquisition noise

- Auto-covariance for a typical imaging system. Notice that the covariance along the horizontal direction, which is a characteristic often observed in CCD cameras



Modeling image noise

- Additive noise model

$$\text{Random noise } n(i,j) \text{ added to pixel value } I(i,j)$$

$$\hat{I}(i,j) = I(i,j) + n(i,j)$$

- Signal-to-noise ratio (SNR), often expressed in *decibel*

$$SNR = \frac{\sigma_s}{\sigma_n}$$

$$SNR_{dB} = 10\log_{10}\left(\frac{\sigma_s}{\sigma_n}\right)$$

- 20 dB means $\frac{\sigma_s}{\sigma_n} = 100$

Department of Computer Engineering
University of California at Santa Cruz

Modeling image noise

- Gaussian noise -white Gaussian, zero-mean stochastic process
  - White – *n(i,j)* independent in both space and time
  - Zero-mean – $\bar{I}(i,j) = 0$
  - Gaussian - *n(i,j)* is random variable with distribution

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

- Impulsive noise – also called peak, spot, or salt and pepper noise, caused by transmission errors, faulty CCD sites, etc.

$$I_{sp}(i,j) = \begin{cases} I(i,j) & x < l \\ I_{\min} + y(I_{\max} - I_{\min}) & x \geq l \end{cases}$$

$x, y \in [0,1]$ are two uniformly distributed random variables

Modeling image noise

- Example: Gaussian noise and salt and pepper noise, *l=0.99*

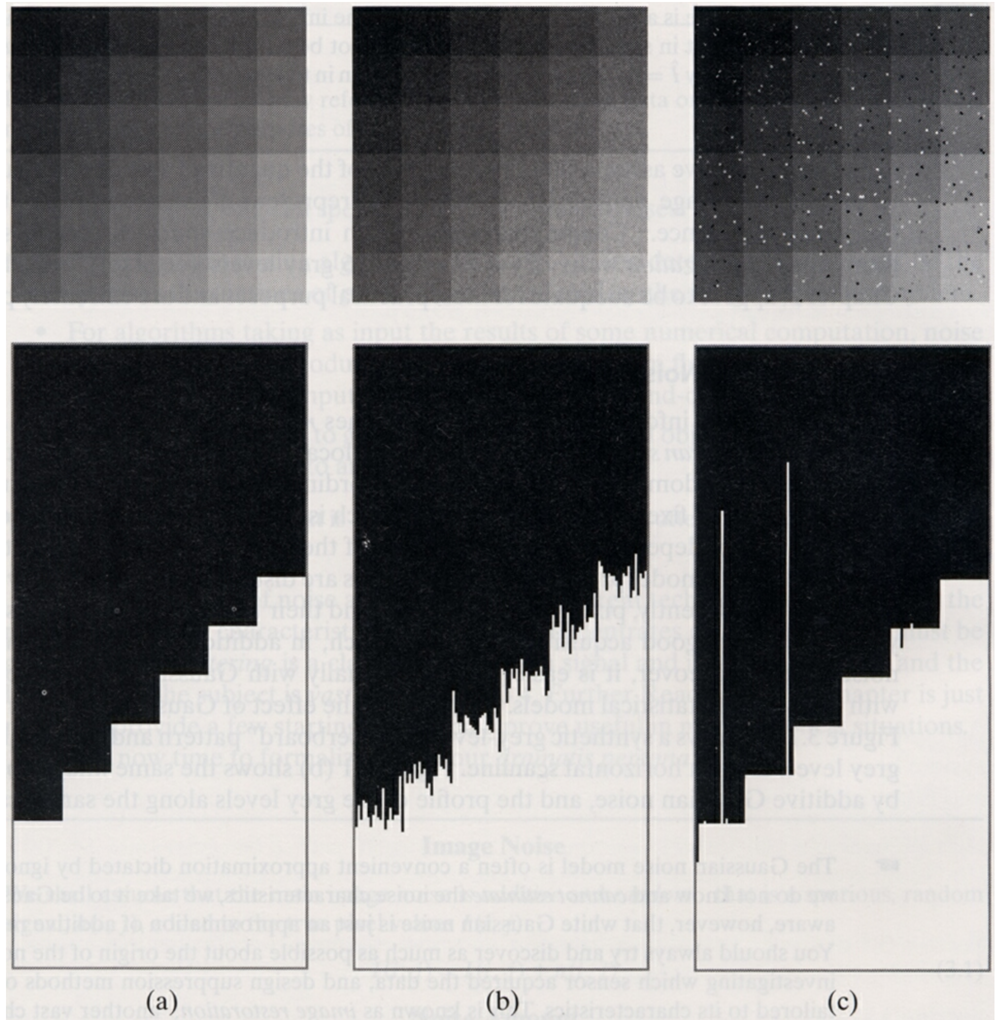


Figure 3.1 (a) Synthetic image of a 120 × 120 grey-level "checkerboard" and grey-level profile along a row. (b) After adding zero-mean Gaussian noise ($\sigma = 5$). (c) After adding salt and pepper noise (see text for parameters).

Department of Computer Engineering
University of California at Santa Cruz

Filtering noise - denoise

- Goal: recover $I(i,j)$ from $\hat{I}(i,j) = I(i,j) + n(i,j)$
- Methods: linear filtering and nonlinear filtering
- Linear filtering: replace the original pixel by the weighted sum of its neighboring pixel. The weights are the filter coefficients

$$I : original\ image$$

$$A : filter\ kernel, size\ m \times m$$

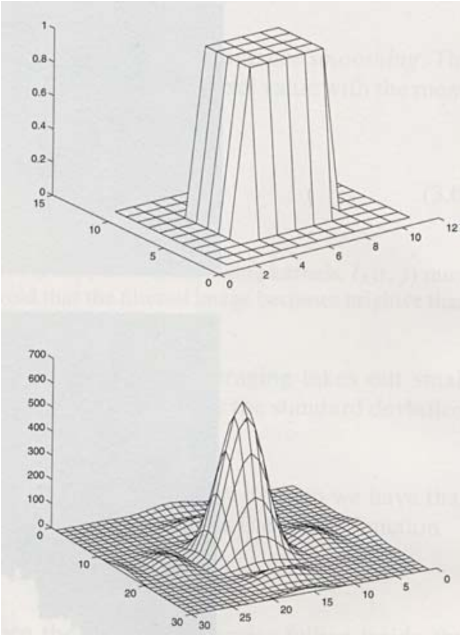
$$I_A(i,j) = I * A = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} A(h,k) I(i-h, i-k)$$

Department of Computer Engineering
University of California at Santa Cruz

Mean filter – smoothing by averaging

- The basic assumption is that the noise has higher frequency and the signal has lower frequency. Noise can be canceled by low-pass filtering
- A averaging filter with m=5 is

$$A_{avg} = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



- Why does it work ?
  - Explanation in spatial domain: reduce standard deviation by 5
  - Explanation in frequency domain: suppress high frequency components since $F(I * A) = F(I)F(A)$

Gaussian filter

- The Fourier transform of a Gaussian kernel is also Gaussian, better low-pass filter than averaging filter

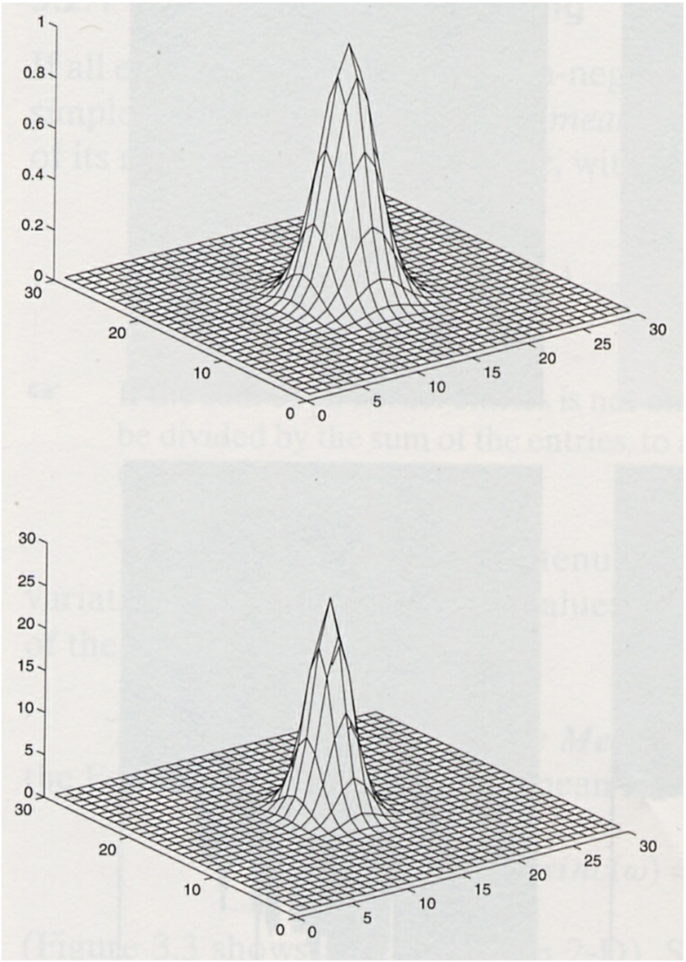
$$G(h,k) = e^{-\frac{h^2+k^2}{2\sigma^2}}$$

- Gaussian kernel is separable, which means 2D Gaussian filtering can be implemented as two 1D Gaussian filtering

$$I_G = I * G = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} G(h,k) I(i-h, i-k)$$

$$= \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} e^{-\frac{h^2+k^2}{2\sigma^2}} I(i-h, i-k)$$

$$= \sum_{h=-m/2}^{m/2} e^{-\frac{h^2}{2\sigma^2}} \sum_{k=-m/2}^{m/2} e^{-\frac{k^2}{2\sigma^2}} I(i-h, i-k)$$



Gaussian filter

- SEPAR_FILTER
  - Build a 1-D Gaussian filter *g*, of width $\sigma_g = \sigma_G$
  - Convolve each row of *I* with *g*, yielding a new image I_r
  - Convolve each column of I_r with *g*, yielding a new image I_G
- Construction of Gaussian filter
  - In order to subtend 98.76% of the area in the Gaussian function, for a given σ, the width of the filter need to be five times of σ , or $w = 5\sigma$
  - Integer filter: Gaussian filter can be implemented efficiently by converting the real numbers into rational numbers with a common denominator: scale the filter so that the smallest components are 1s, replace the other entries with the closest integers

Gaussian filter

- Example
  - Filter image with Gaussian noise
  - Filter image with salt and pepper noise

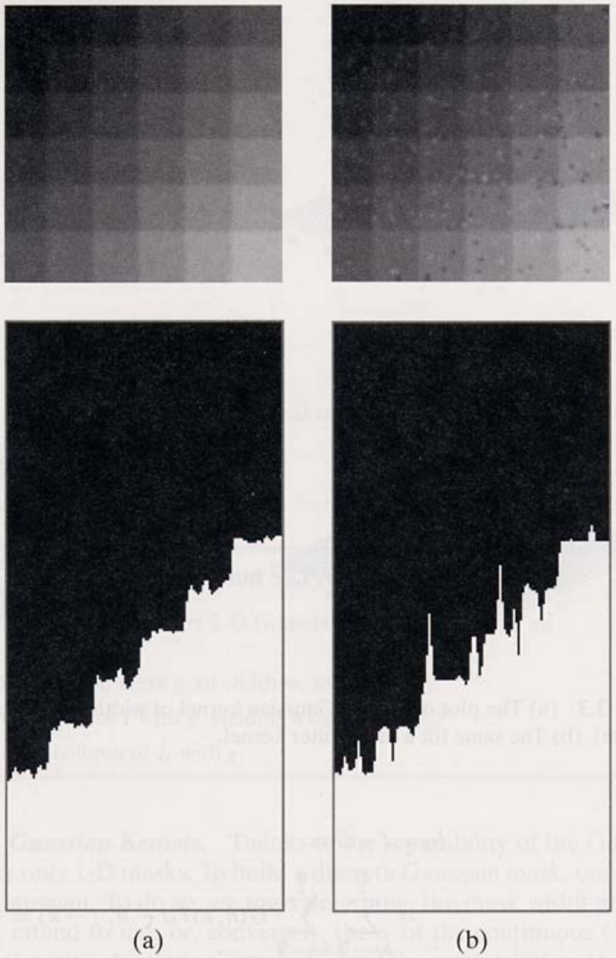


Figure 3.2 (a) Results of applying Gaussian filtering (kernel width 5 pixel, $\sigma = 1$) to the "checkerboard" image corrupted by Gaussian noise, and grey-level profile along a row. (b) Same for the "checkerboard" image corrupted by salt and pepper noise.

Nonlinear filtering

- Median filtering
  - Algorithm MED_FILTER
  - For each pixel *I(i,j)* and its $n \times n$ neighborhood,
    - Sort its neighboring pixels $\{I(i+h, i+k) \mid h,k \in [-n/2, n/2]\}$
    - Assign the median value to *I(i,j)*
  - Example: with window/mask size 3×3

22	17	102	105	106
21	21	100	99	102
19	22	20	102	102
24	21	97	101	104
19	18	101	108	101

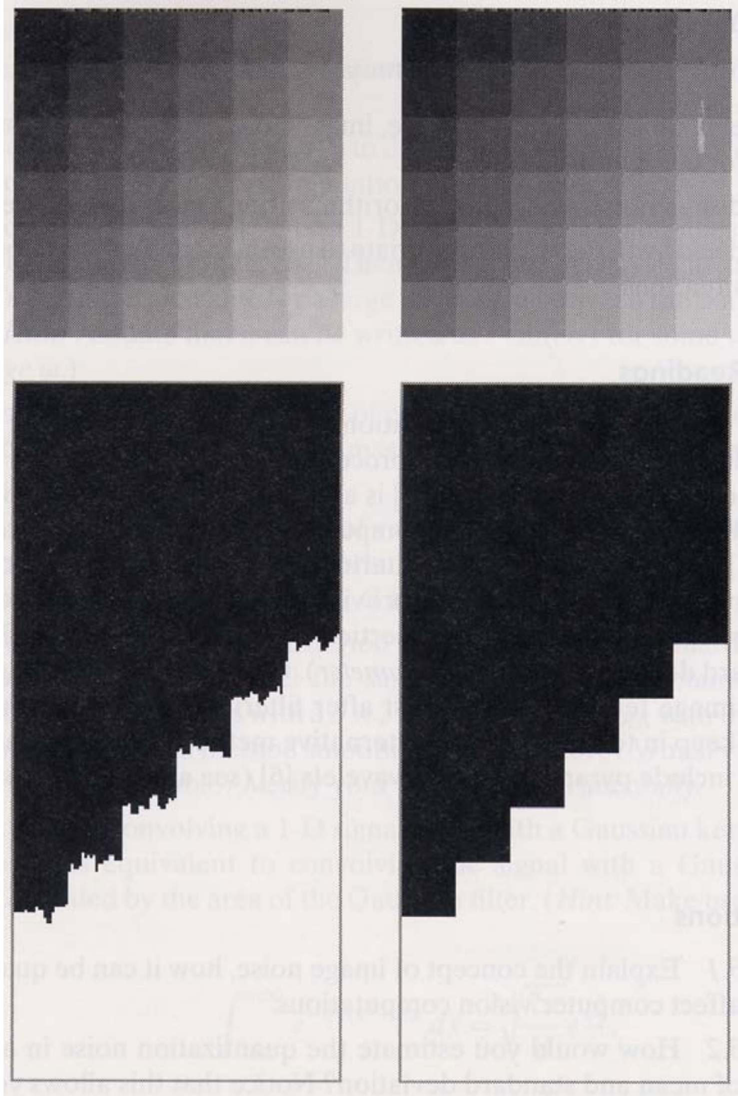
⇒

22	17	102	105	106
21	21	99	102	102
19	21	97	102	102
24	21	97	101	104
19	18	101	108	101

Department of Computer Engineering
University of California at Santa Cruz

Median filtering

- Example with Gaussian noise
  - Preserve discontinuity in the signal.
  - No contribution from pixels with large noise
  - Expensive to compute



(a) (b)

Homework

- Use Matlab to complete the following experiment
  - Read in a gray-level image
  - Add Gaussian noise to the image with $\sigma = 10$
  - Implement
    - 5 by 5 Separable Gaussian filter with $\sigma = 0.8$
    - 5 by 5 Median filter
  - Turn in
    - Matlab code for adding noise and the two filtering algorithms
    - The original image, the image with noise, and the filtering results
    - Derivation of the discrete Gaussian filter