CMPE 8 NAME: ______________________
Fall 2008 Quiz 2

Quiz Time: 12:05–12:45 PM

1. For the dynamic model
$$x_{k+1} = x_k + \frac{1}{10}\left(8 - x_k^3\right)$$

   (a) (5 points) Calculate the fixed point(s) x_*.
   First substitute x_* in for x_k and x_{k+1}:
   $$x_* = x_* + \frac{1}{10}\left(8 - x_*^3\right)$$
   Now, solve for x_*:
   $$0 = \frac{1}{10}\left(8 - x_*^3\right) \quad \Longrightarrow \quad x_*^3 = 8 \quad \Longrightarrow \quad x_* = 2$$
   So, $x_* = 2$ is the only fixed point.

   (b) (6 points) Use a calculator and compute the first 6 elements for two different orbits: one starting from $x_1 = 0$, and the other starting from $x_1 = 4$. So, your answer should be two vectors, each with 6 elements.

   vector 1: (0, 0.8, 1.55, 1.98, 2.00, 2.00)

   vector 2: (4, -1.6, -0.39, 0.42, 1.21, 1.83)

   (c) (4 points) Based on the orbits calculated, make a judgment about whether the fixed point(s) are stable, stable and attractive, or unstable. Justify your choice.

   Both orbits appear to converge back to the fixed point $x_* = 2$, suggesting that this fixed point is stable and attractive.

2. (4 points) Provide a model that has no fixed points. Your answer should be in the form $x_{k+1} = f(x_k)$, and you provide the function f.
$$x_{k+1} = x_k + 1$$

3. (4 points) Provide a model that has infinitely many fixed points. Again, your answer should be in the form $x_{k+1} = f(x_k)$, and you provide the function f.
$$x_{k+1} = |x_k|$$

4. **Bonus**: (10 points) A dynamic model of a cruise-controlled car's speed is

$$v_{k+1} = v_k + \frac{\Delta}{m}[-bv_k + u_k],$$

where the cruise controller is $u_k = c_1[v_{\text{des}} - v_k], \quad c_1 > 0$. Calculate the fixed point v_* assuming constant desired speed v_{des}.

First substitute u_k into the speed equation:

$$v_{k+1} = v_k + \frac{\Delta}{m}[-bv_k + c_1[v_{\text{des}} - v_k]].$$

Next, substitute v_* in for v_k and v_{k+1}:

$$v_* = v_* + \frac{\Delta}{m}[-bv_* + c_1[v_{\text{des}} - v_*]].$$

Now, solve for v_*. First, cancel the v_* terms on both sides and simplify a bit:

$$0 = \frac{\Delta}{m}[-bv_* + c_1 v_{\text{des}} - c_1 v_*].$$

Next, multiply both sides by the constant m/Δ, collect terms that have v_* on one side of the equation, and collect all other terms on the other side of the equation:

$$v_*(b + c_1) = c_1 v_{\text{des}}.$$

Solving for v_* gives the solution:

$$v_* = \frac{c_1 v_{\text{des}}}{b + c_1}.$$

How close does the actual fixed point v_* get to the desired value v_{des}? What choice for c_1 reduces this discrepancy? These are important questions in determining how good the controller is, and we will address these in more detail in later chapters.

5. **Bonus**: (5 points) Prove analytically that the fixed point(s) computed in problem 1(a) have the stability behavior that you observed in part (b), and chose in part (c). Hint: this involves taking a derivative.

$$f(x) = x + (8 - x^3)/10 \quad \text{and} \quad \frac{df(x)}{dx} = 1 - \frac{3}{10}x^2.$$

(The derivative was provided during the quiz). As discussed in class, the fixed point is stable and attractive since $df(2)/dx = -0.2$ which has absolute value less than 1.