CMPE 8 NAME: Solutions

Fall 2008 Quiz 3

Test Time: 12:00–12:40 PM

1. (10 points) The pendulum has the rotational angle variable θ, and the model for the pendulums motion has the form

$$\theta_{k+1} = f(\theta_k, \theta_{k-1}), \quad k = 2, 3, \ldots$$

Write the **discrete-time dynamic model of the pendulum**, including the affects of gravity and of friction in the joint. Your answer should include parameters $(l, b, I, \Delta) =$ (length of pendulum, friction coefficient, moment of inertia, sample period).

$$f(\theta_k, \theta_{k-1}) = 2\theta_k - \theta_{k-1} - \Delta^2\left(\frac{g}{l}\right)\sin(\theta_k) - \Delta\left(\frac{b}{I}\right)(\theta_k - \theta_{k-1})$$

2. (10 points) For the dynamic model of the pendulum above, compute the equilibrium anlge(s) θ_*. Show all steps to get full credit.

$$\cancel{\theta_*} = \cancel{2}\theta_* - \cancel{\theta_*} - \Delta^2\left(\frac{g}{l}\right)\sin(\theta_*) - \Delta\left(\frac{b}{I}\right)\cancel{(\theta_* - \theta_*)}$$

$$\Delta^2\left(\frac{g}{l}\right)\sin(\theta_*) = 0 \implies \theta_* = 0, \pi$$

or

$$\theta_* = k\pi$$

$$k = 0, \pm 1, \pm 2, \ldots$$

3. (10 points) Write a function in Matlab whose output is a plot of the pendulum angle orbit $(\theta_1, \theta_2, \theta_3, \ldots)$ versus time $(t_1, t_2, t_3, \ldots)$, with the pendulum model starting from $\theta_1 = \theta_2 = 0.1$, with (l, b, I) as function input parameters, and with start time $t_1 = 0$, end time $t_N = 20$, and sample period $\Delta = 0.01$.

```
function mypend(L,b,I)
t=[0:0.01:20];
th(1)=0.1;
th(2)=0.1;

for k=2:length(t)-1
    th(k+1)= 2*th(k) - th(k-1) - (0.01^2)*9.81/L*sin(th(k)) - 0.01*b/I*(th(k)-th(k-1));
end
plot(t,th)
```

4. **Bonus:** (5 points) The pendulum model with **torque control input** u_k has the form

$$\theta_{k+1} = f(\theta_k, \theta_{k-1}) + \Delta^2 u_k, \quad k = 2, 3, \ldots \tag{1}$$

where $f(\theta_k, \theta_{k-1})$ is the pendulum model as before. With $\theta = \pi$ as the upright (inverted) angle, show that the inverted angle is a fixed point for the model (1), when the feedback torque $u_k = c_1(\pi - \theta_k)$ is used. Show all work.

$$\theta_{k+1} = 2\theta_k - \theta_{k-1} - \Delta^2\left(\frac{g}{l}\right)\sin(\theta_k) - \Delta\left(\frac{b}{I}\right)(\theta_k - \theta_{k-1}) + \Delta^2 c_1(\pi - \theta_k)$$

eq. $\rightarrow$ $0 = -\Delta^2\left(\frac{g}{l}\right)\sin(\theta_*) + \Delta^2 c_1(\pi - \theta_*)$

$\theta_* = \pi$ satisfies this equation

($\sin(\pi) = 0$ and $(\pi - \pi) = 0$)

5. **Bonus:** (2 points) If $c_1 > 0$ makes the inverted angle stable and attractive, what would likely be the stability of the inverted angle fixed point with $c_1 < 0$?

unstable

CMPE 8 NAME: ________________
Fall 2008 Quiz 4

Test Time: 12:05–12:45 PM. Be sure to write CLEARLY!

1. (5 points) Write the Matlab commands to generate a time vector `t` that starts with 10, ends with 20, and has a sample period of 0.5.

   ≫ t=[10:0.5:20];

2. (6 points) For the time vector `t` defined above, what are: a) `t(3)`, b) `t(8)` and c) the length of the vector?

   a) 11

   b) 13.5

   c) 21

3. (3 points) Evaluate each of your team members performance by filling out the following table. Use the following ranking:

   1=excellent, 2=very good, 3=good, 4=less than satisfactory, 5=totally lame

Team Member Name	Ranking
me	1

Team Name:

4. (5 points) Write down the dynamic model for a robot's translational position x and rotational position θ. Included in the model are the constant sample period Δ, translational speed u_k and rotational speed v_k.

$$\begin{aligned} x_{k+1} &= x_k + \Delta u_k \cos(\theta_k) \\ \theta_{k+1} &= \theta_k + \Delta v_k \end{aligned} \tag{1}$$

5. (5 points) The wall-following control is given by

$$u_k = u_{\text{nom}}, \quad v_k = k_p\left(x_k - d_{\text{sep}}\right) + k_d \frac{x_k - x_{k-1}}{\Delta}, \tag{2}$$

where k_p, k_d, d_{sep} and u_{nom} are positive constants. Substitution of (2) into (1) gives the **closed-loop dynamic model of the robot.**

Write down the closed-loop dynamic model of the robot.

$$x_{k+1} = x_k + \Delta u_{nom} \cos(\theta_k)$$
$$\theta_{k+1} = \theta_k + \Delta\left[k_p(x_k - d_{sep}) + \frac{k_d}{\Delta}(x_k - x_{k-1})\right]$$

6. (5 points) Recall that the **control objective** is for $(x_*, \theta_*) = (d_{\text{sep}}, \pi/2)$ to be a stable and attractive equilibrium point of the closed-loop dynamic model, while the robot moves at a constant speed u_{nom} in the y direction.

Show that $(x_*, \theta_*) = (d_{\text{sep}}, \pm\pi/2)$ are the two distinct equilibrium points for the closed-loop dynamic model of the robot.

$$\cancel{x_*} = \cancel{x_*} + \Delta u_{nom} \cos(\theta_*)$$
$$\cancel{\theta_*} = \cancel{\theta_*} + \Delta\left[k_p(x_* - d_{sep}) + \frac{k_d}{\Delta}(\cancel{x_*} - \cancel{x_*})\right]$$

$$\rightarrow \cos(\theta_*) = 0 \longrightarrow \theta_* = \pm \pi/2$$

$$\rightarrow 0 = \Delta k_p (x_* - d_{sep}) \longrightarrow x_* = d_{sep}$$