Review problems for AMS 206

1. Suppose we are trying to learn about how people intend to vote in California in the fall presidential election. Asking 10 nearby people finds that only one plans to vote for the Republican candidate. We plan to randomly survey 100 people in California who plan to vote in the general election.

   (a) What would be a conjugate prior for the probability a Californian plans to vote for the Republican candidate that fully incorporates this information?

   (b) Because this initial sample isn't a random sample from the whole state, suppose we only want to give it half as much credibility. Now what would our prior be?

   (c) Our full survey finds that 38 out of 100 people plan to vote for the Republican candidate. Using the prior from part (b), what is our posterior distribution? (You do not need to re-derive the formula.)

   (d) Derive the posterior predictive distribution for whether we think an individual Californian will vote for the Republican candidate.

2. Suppose we will be observing a **single** draw from a normal distribution with unknown mean μ and variance one, i.e., $Y \sim N(\mu, 1)$, and suppose we have a normal prior for μ with mean zero and variance 3, i.e., $\mu \sim N(0, 3)$. Derive the prior predictive distribution, i.e., find our marginal distribution for Y. You should be able to get it into the form of a recognizable distribution and to identify that distribution.

3. In the morning you pull a pair of pants out of your drawer and put them on. You remember that the last time you wore them you left either a 5 dollar bill or a 20 dollar bill in the back pocket (and no other bills), but you can't remember which, and think that either case is equally likely. During the day, a friend repays you a 5 dollar bill that you had lent them the day before, and you put it the same back pocket. Later that day, you go to buy a coffee, and randomly pull one of the two bills out of your back pocket (with equal probability). You find that you have pulled out a 5 dollar bill. Use Bayes' Theorem to find the probability that you now have a 20 dollar bill in your pocket (or equivalently, that you started the day with a 20 dollar bill).