

Review problems for AMS 206

1. Suppose we are trying to learn about how people intend to vote in the upcoming Democratic presidential primary. Asking 10 nearby people finds that only one plans to vote for Hillary Clinton. We plan to randomly survey 100 people in California who plan to vote in the Democratic primary.
  - (a) What would be a conjugate prior for the probability a Democratic Californian plans to vote for Hillary that fully incorporates this information?
  - (b) Because this initial sample isn't a random sample from the whole state, suppose we only want to give it half as much credibility. Now what would our prior be?
  - (c) Our full survey finds that 38 out of 100 people plan to vote for her. Using the prior from part (b), what is our posterior distribution? (You do not need to re-derive the formula.)
  - (d) Derive the posterior predictive distribution for whether we think an individual Democrat will vote for Hillary in the primary.

2. Practice with indicator functions/space restrictions: Suppose we are doing rejection

sampling, and our proposal function is  $G(\theta) = \begin{cases} 0.2 & \text{if } 0 \leq \theta \leq 0.2 \\ 0.6 & \text{if } 0.2 < \theta \leq 0.5 \\ 2.3 & \text{if } 0.5 < \theta \leq 0.7 \\ 1.4 & \text{if } 0.7 < \theta \leq 1.0 \\ 0 & \text{otherwise} \end{cases}$

i.e.,  $G(\theta) = 0.2I_{\{0 \leq \theta \leq 0.2\}} + 0.6I_{\{0.2 < \theta \leq 0.5\}} + 2.3I_{\{0.5 < \theta \leq 0.7\}} + 1.4I_{\{0.7 < \theta \leq 1\}}$ .

- (a) Find the sampling density  $g(\theta)$ .
- (b) What is the probability a proposal will be drawn between 0.4 and 0.6?
- (c) What is the marginal probability a proposal will be accepted?