AMS 7 12 Feb 09 ①

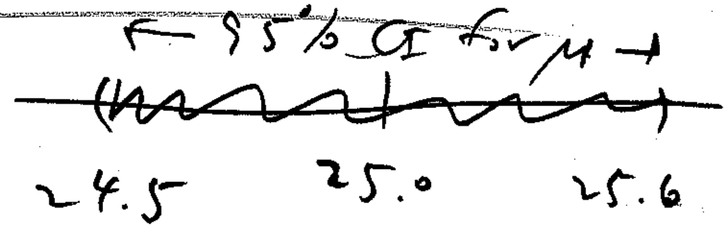
this time: inference for proportions; next time: hypothesis tests

lab 3 due by 5pm tomorrow

homework 3 due Tue 24 Feb

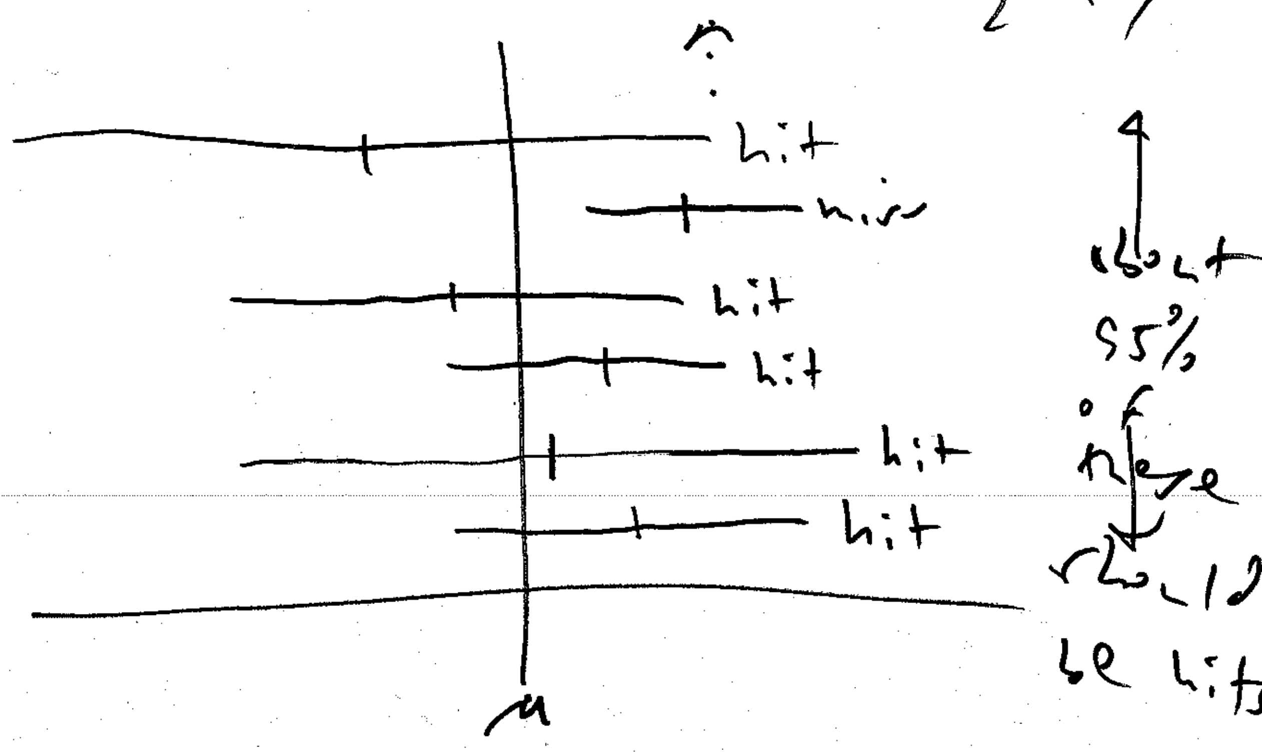
holiday next Mon: no labs, disc. sec or office hrs; if you usually have lab and/or section on Mon, you need to go to another lab and/or section next week

meaning of CIs



does this mean that $P_f(24.5 < \mu < 25.6) = 95\%$? no

f ← relative frequency



(relevant) pop ~~all lab rats~~ (similar (ask bidogists)) — sample: the observed lab rats — imag data: possible $\hat{p}$s ②

1 = Y
0 = N

left? (actual) like SRS : IID → left?

$N = ?$ (big) $\begin{pmatrix} 1s \\ \cancel{x} \\ 0s \end{pmatrix}$ — $\begin{pmatrix} 1s \\ 0s \end{pmatrix}$ $n = 12$ — $\begin{pmatrix} 83\% \\ 75\% \\ \vdots \end{pmatrix}$ $M = \infty$

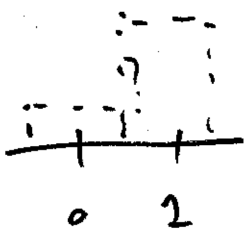
mean $\bar{y} = \hat{p} = \frac{10}{12}$ "p hat" $\doteq 83\%$ $= 0.83$

mean $\mu = p = ?$

SD $\sigma = \sqrt{p(1-p)} = ?$

hyp. IID

p hist



$\begin{pmatrix} \; \end{pmatrix}$ $n = 12$

mean $\hat{p} = ?$ (ex. $\frac{9}{12} = 75\%$)

long run mean: expected value of $\hat{p} = p$ ~~est.~~ $\boxed{1}$

est. long run SD: standard error of $\hat{p} \doteq 11\%$ $\boxed{2}$

long run hist $\boxed{3}$

$\boxed{1}$ EV of $\hat{p} = E_{IID}(\hat{p}) = E_{IID}(\bar{y}) = \mu = p$

$\boxed{E_{IID}(\hat{p}) = p}$ math fact

$\boxed{2}$ est. SE of $\hat{p} = \hat{SE}_{IID}(\hat{p})$

$= \hat{SE}_{IID}(\bar{y}) = \frac{\hat{\sigma}}{\sqrt{n}}$

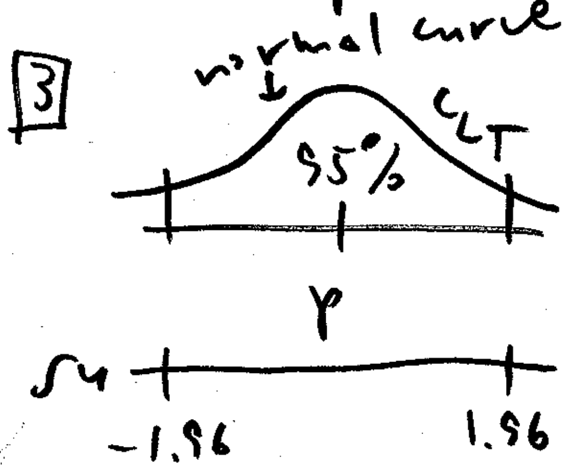
earlier math fact: if a pop has only 2 values in it,

$\sigma = \left(\begin{matrix}\text{larger} \\ \text{value}\end{matrix} - \begin{matrix}\text{smaller} \\ \text{value}\end{matrix}\right)\sqrt{\left(\begin{matrix}\text{proportion of} \\ \text{larger values}\end{matrix}\right) \cdot \left(\begin{matrix}\text{prop. of} \\ \text{smaller values}\end{matrix}\right)}$

inferential summary

pop	unknown pop. quantity of interest	p = pop % of rats similar to these that would turn left toward food
sample	estimate of p	$\hat{p} = 83\%$
imag data	give or take for $\hat{p}$ as est. of p	$\widehat{SE}_{IID}(\hat{p}) \doteq 11\%$
	approximate 95% CI for p	$\hat{p} \pm 1.96\,\widehat{SE}(\hat{p}) = (61\%, 100\%)$ (truncated)

$\boxed{3}$



long run hist of $\hat{p} = \bar{y}$ if n large

so here larger value = 1

smaller value = 0

& SD of a 0-1 pop is

$$\sigma = \sqrt{p(1-p)}$$

math fact with 0-1 pop

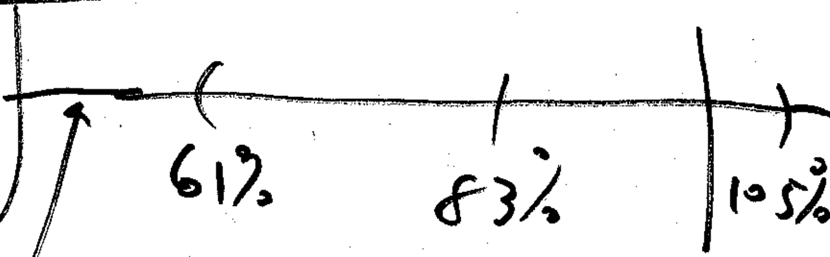
so $\widehat{SE}_{IID}(\hat{p}) = \dfrac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$$= \sqrt{\frac{(0.83)(0.17)}{12}} \doteq 0.108 \doteq 10.8\% \doteq 11\%$$

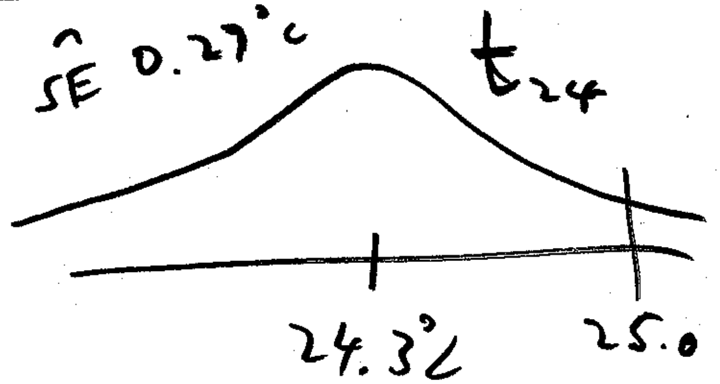
$\bar{y} \pm t^{0.95}_{n-1} \, \hat{SE}_{IID}(\bar{y})$ continuous outcome

$\hat{p} \pm 1.96 \, \hat{SE}_{IID}(\hat{p})$ 0-1 outcome (approximate)

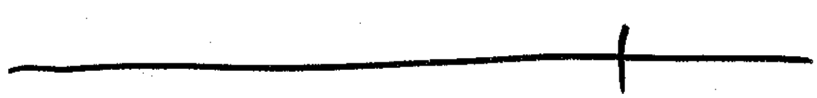
$83\% \pm (2)(11\%)$



$p_0 = 50\%$ is not in 95% CI, so the difference between 83% ($\hat{p}$) & 50% (p_0) is statsig (is probably real)



long run hist of $\bar{y}$ if null true, accounting for uncertainty in σ ("t statistic")



$$2.59 = \frac{+0.7}{0.27} = \frac{25.0 - 24.3}{0.27} = t$$