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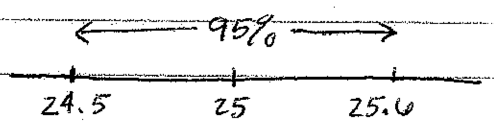
- <u>Not real</u>: easy to attribute to unlucky random sampling
- <u>Real</u>: hard to attribute to unlucky random sampling (statistically significant)

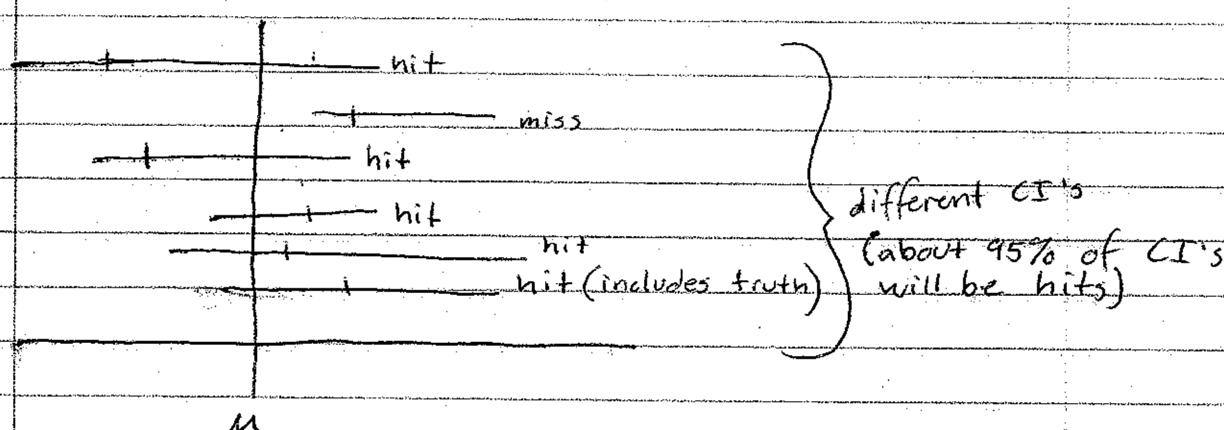
Statistically Significant = difference between $\bar{y}$ and μ hard to attribute to unlucky random sampling = difference (probably) real.

- <u>Meaning of Confidence Intervals</u>



Does this mean that $P_r(24.5 < \mu < 25.6) = 95\%$? <u>No</u>

↑ relative frequency



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- Lab rat case study

Population	Sample	Imaginary
all similar lab rats	observed lab rats	possible $\hat{p}$

left?

N = ? Big

[1's or 0's] $\xrightarrow{\text{like SRS}}$ [1's or 0's] n = 12

hypothetical

mean $\mu = ? = p$

SD $\sigma = \sqrt{p(1-p)}$

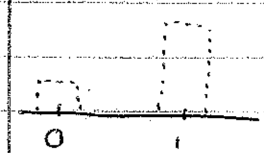
mean $\bar{y} = \hat{p} = \frac{10}{12} = .83$

[83%, 75%] M = ∞

[ ] n = 12

mean $\hat{p}$ = ?

ex. 75%



- est. long run mean = p (expected value of $\hat{p}$) $\boxed{1}$
- est. long run SD = 11% (standard error of $\hat{p}$) $\boxed{2}$
- long run hist. $\boxed{3}$

Inferential Summary

pop.	unknown pop. quantity of interest	p = pop. % of rats similar to those that would turn left for food
sample	estimate of p	$\hat{p}$ = 83%
↑ imaginary data ↓	give or take for $\hat{p}$ as estimate of p	$\hat{SE}_{IID}(\hat{p}) = 11\%$
	95% CI for p	$\hat{p} \pm 1.96\, SE_{IID}(\hat{p}) = (61\%, 100\%)$ (truncated)

$\boxed{1}$ EV of $\hat{p} = E_{IID}(\hat{p}) = E_{IID}(\bar{y}) = \mu = p$

$$\boxed{EV_{IID}(\hat{p}) = p}$$

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$\boxed{2}$ SE of $\hat{p}$ = $\hat{SE}_{IID}(\hat{p}) = \hat{SE}_{IID}(\bar{y}) = \sigma/\sqrt{n}$

$$\sigma = (\text{larger value} - \text{smaller value})\sqrt{\begin{pmatrix}\text{proportion}\\ \text{of larger}\\ \text{values}\end{pmatrix}\begin{pmatrix}\text{proportion}\\ \text{of smaller}\\ \text{values}\end{pmatrix}}$$

$$\sigma = (1-0)\left(\sqrt{(p)(1-p)}\right)$$

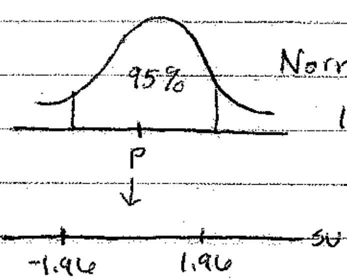
* b/c there are only 2 values in the pop.

So, $\hat{SE}_{IID}(\hat{p}) = \dfrac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$

$$= \sqrt{\frac{(.83)(.17)}{12}} = .108 = 10.8\% \approx 11\%$$

- $\bar{y} \pm t^{.95}_{n-1}\ \hat{SE}_{IID}(\bar{y})$ w/continuous outcome (appoximate)

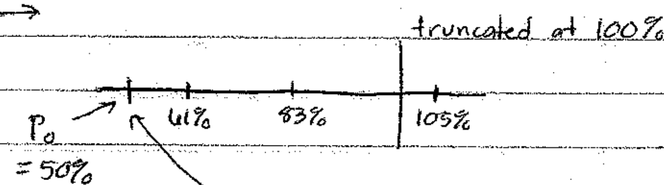
$\boxed{3}$



Normal curve from CLT

long run histogram of $\hat{p} = \bar{y}$ if n is large

$83\% \pm 2(11\%) \rightarrow$



- Predicted that p would be 50%, not in 95% CI
- Difference between 83% ($\hat{p}$) and 50% (p_0) is statistically significant (is probably real)

- Hypothesis

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null hypothesis (H_0)	$\mu = 24.3°C$	Theory $\boxed{1}$
alternative hypothesis (H_A)	$\mu \neq 24.3°C$	Theory wrong $\boxed{2}$

- try null, see how well it fits data, compare w/actual data. If difference is large, H_0 is bad, alternative is favored

$\boxed{1}$ difference between 24.3 and $\bar{y} = 25$ is due to unlucky random sampling

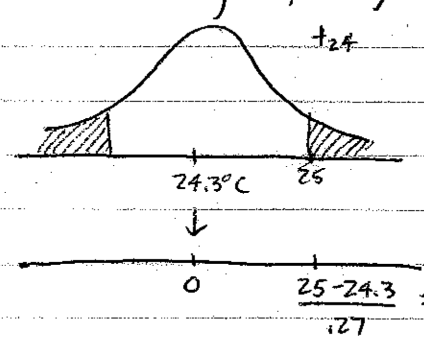
$\boxed{2}$ difference between 24.3 and 25 is real

- Try null, see if discrepancy between:

  * how data came out vs. ** how dada should have come out if null true

  - is large; if yes, favor alternative hypothesis (reject null)
  - is not large; favor null ("fail to reject null")

- long run histogram of $\bar{y}$ if null were true (accounting for uncertainty of σ)



$\frac{25-24.3}{.27}$ = t statistic

= 2.59